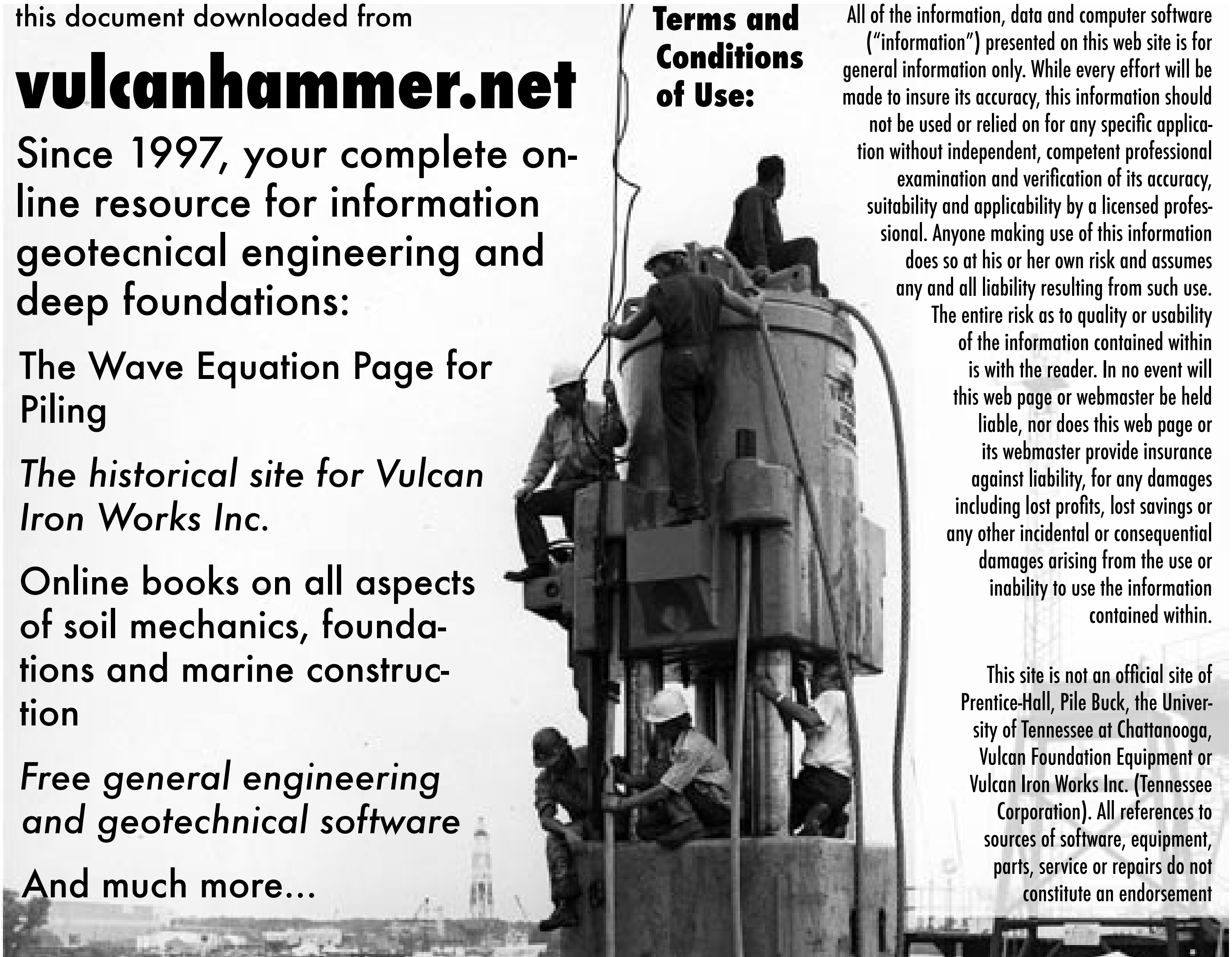

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A Frequency-Dependent Soil Propagation Model

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ABSTRACT

Vibration analyses of advanced technology facilities typically must consider frequency as well as amplitude of vibration. A soil propagation model is proposed which will allow the use of site-specific, measurable, frequency dependent attenuation characteristics. A method is given which allows *in-situ* determination of those frequency-dependent properties. This approach is applied to the estimation of setback distances for various items of construction equipment at a particular site.

Keywords: vibration propagation, attenuation, construction vibrations, soil vibration

1. INTRODUCTION

Vibration amplitude can be quantified in terms of displacement, velocity or acceleration; each can be stated in either time or frequency domain. Issues related to vibration-sensitive facilities—such as semiconductor production plants—are generally treated in the frequency domain.[*] Unfortunately, the literature of simplified approaches to vibration propagation in soil is very much limited to the time domain, since the areas of interest have generally been construction and building damage. These methods alone do not lend themselves particularly well to assessment in the frequency domain.

This paper presents a discussion of the frequency-dependent aspects of vibration propagation, including a propagation model which takes frequency content into consideration, and gives a method by which *in-situ* frequency-dependent propagation properties of a site can be measured and used in propagation calculations.

2. PROPAGATION MODELS

2.1 Basic Theory

Vibrations propagate from a source on or near the ground surface through the ground to a distant vibration-sensitive receiver predominantly by means of Rayleigh (surface) waves and secondarily by body (shear and compressional) waves. The amplitude of these waves diminishes with distance from the source. This attenuation is due to two factors: expansion of the wave front (geometrical attenuation) and dissipation of energy within the soil itself (material damping). The rate of geometrical attenuation depends upon the type of wave and the shape of the associated wave front. Material damping is generally thought to be attributable to energy loss due to hysteresis, perhaps caused by internal sliding of soil particles. The amount of material damping that occurs is a function of the vibration amplitude; we will limit our discussions to what are generally considered low-amplitude cases.

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* It is common practice to represent vibration spectra for vibration-sensitive facilities in terms of one-third octave bands of frequency. For example, see Colin G. Gordon's “Generic criteria for vibration-sensitive equipment,” *Proceedings of SPIE Conference on Vibration Control and Metrology*, pp. 71-85, San Jose, CA (November 1991), or Hal Amick's “On Generic Vibration Criteria for Advanced Technology Facilities,” *Journal of the Institute of Environmental Sciences*, pp. 35-44, (Sept/Oct 1997).

Material damping in soil is a function of many parameters, including soil type, moisture content and temperature. Clays tend to exhibit higher damping than do sandy soils.[1] Wet sand attenuates less than dry sand because the pore water between sand particles carries a significant portion of compressional energy and thus does not subject compressional waves to as much attenuation by friction damping. Propagation of Rayleigh waves is insensitive to the presence or absence of water.[2] Frozen soil attenuates less than thawed soil.[3]

The general equation modeling propagation of ground vibration from point "a" (a location at distance r_a from the source) to point "b" (a location at distance r_b from the source) may be stated in the form of Equation (1),

$$v_b = v_a \left(\frac{r_a}{r_b} \right)^{\gamma} e^{\alpha (r_a - r_b)} \tag{1}$$

where γ is a coefficient dependent upon the type of propagation mechanism and α is a material damping coefficient.

Theoretical radiation models based upon half-space formulation have been used to determine γ corresponding to various propagation models in idealized cases. This form of attenuation can also be expressed in terms of decibels per doubling of distance. Several commonly accepted values of γ are shown in Table 1 for a number of types of sources and waves.

Most settings involve surface (or near-surface) sources and receivers, and Rayleigh wave propagation is the most common. Even when the actual vibration source is below the surface—as with pile driving—Rayleigh waves are formed within a few meters of the point on the surface directly above the source, and the propagation can be modeled in terms of Rayleigh waves.[4]

2.2 Traditional Assessment in Time Domain

There are two principal forms in which investigators have fit Equation (1) to observed data. One approach is to neglect damping attenuation and fit geometric attenuation curves to field data. The other approach assumes Rayleigh wave propagation and fits material damping curves to measured data.

In the first approach, one sets $\alpha = 0$ and assumes that attenuation follows a straight line on a log-log plot of velocity amplitude as a function of distance. In this case, γ is the slope of that line in decades per decade. Investigators have found values of γ between 0.8 and 1.7. Table 2 summarizes some of the published values of γ.

Using the other approach, one sets $\gamma = 0.5$ and selects a value of α based upon soil type.

Table 3 summarizes some of the published values of α, which range from 0.012 to 0.134 ft^{-1}. Using this approach, one assumes that attenuation is independent of frequency.

3. FREQUENCY-DEPENDENT MATERIAL ATTENUATION

Barkan and Dowding observe that a soil's material damping provides a specific amount of attenuation per wavelength.[3,4] This is consistent with observations for many propagation media. It is typical to assume a damping material property that is constant for a wide range of frequencies. There are many numeric quantities used to denote damping, including loss factor η,[5] damping ratio ζ,[5] resonance amplification factor Q,[5] or what Richart, Hall and Woods define as the "specific dissipation function" 1/Q.[2] When a "ringing" vibrating medium is undergoing decay due to damping, the ratio of successive cycles δ, the "log decrement," provides another measure of material damping. These terms are all related, as defined by Equation (2).[5]

$$\eta = 2\zeta = \frac{\delta}{\pi} = \frac{1}{Q} \tag{2}$$

Richart, Hall and Woods define the relationship between α and these material coefficients as shown in Equation (3),[2]

$$d = \frac{2pca}{w} = l\,a = \frac{ca}{f} \tag{3}$$

where c is the wave velocity, λ is the wavelength, and ω is the circular frequency (ω=$2\pi f$). This can be restated in terms of loss factor, as in Equation (4).

$$a = \frac{hpf}{c} \tag{4}$$

For a particular soil deposit, we can assume that both η and c represent constant soil *properties*, therefore a quantity made up of the ratio of the two, $r = \frac{h}{c}$, can also be considered a property of that soil.* Thus, a new relationship can be used to define α:

$$a = rpf \tag{5}$$

Woods and Jedele[6] have proposed a classification of earth materials by attenuation coefficient. Dowding[4] summarizes this in terms of ranges of α at 5 Hz and 50 Hz, but if one uses Equation (5) as a definition of α, one can arrive at a tabulation of ρ as a function of earth material type. This is given in Table 4. The constant ρ can be used in propagation models of the form given in Equation (6).

$$\frac{v_b}{v_a} = \left(\frac{r_a}{r_b}\right)^{g} e^{rpf\,(r_a - r_b)} \tag{6}$$

The vibration attenuation (in dB) between points "a" and "b" can be stated in the form of Equation (7).

$$Attenuation = VL_b - VL_a = 20\log(v_b / v_a) \tag{7}$$

We can rewrite Equation (1) to define attenuation of Equation (7).

$$Attenuation = A_\gamma + A_\alpha \tag{8}$$

where

$$A_\gamma = 20 \times \gamma \times \log_{10}\left(\frac{r_a}{r_b}\right), \text{ and} \tag{8a}$$

$$A_\alpha = 8.68 \times \alpha \times (r_a - r_b)\,, \tag{8b}$$

* An advantage to the use of ρ, rather than η and c, is that ρ is itself a measurable quantity and does not require the measurement or estimation of c.

both terms being expressed in decibels.

4. MEASURING SITE-SPECIFIC ATTENUATION PROPERTIES

When α is traditionally determined from time-domain field measurements, it is generally necessary to obtain measurements of a single activity at several distances. It is difficult to obtain these data at great distance without "contamination" by ambient vibrations. However, one may obtain the frequency-independent damping characteristics (such as ρ or the loss factor η) for use in frequency-dependent analysis from a pair of spectra obtained simultaneously at two distances from a source. These distances need not be great, just enough to be assured that the wave propagation mechanism is via Rayleigh waves.

At one representative site, vibrations were measured at 50 ft and 100 ft from two vibration sources: a scraper and a vibratory compactor operating at two different frequencies (27 and 42 Hz). Linear average, constant-bandwidth FFT spectra were obtained during a single passage of each piece of equipment.

One-third octave band spectra were synthesized, and the transfer functions calculated as the difference (in decibels) between the two spectra. Figure 1 shows the transfer function data measured for the scraper as open circles. At frequencies above 50 Hz, the scraper data were contaminated by ambient vibrations and were neglected in the subsequent analysis. The data from the two one-third octave bands with compactors data, 25 Hz and 40 Hz, are shown as squares. Assuming that the propagation was due to Rayleigh waves, one can determine ρ by carrying out the following steps at each frequency:

- Subtract scalar A_g (3 decibels)—calculated using Equation (8a) for γ=0.5—from the transfer function, leaving spectrum A_a.
- Divide each frequency component of A_α by $8.68(r_b - r_a)$—from Equation (8b), obtaining α as a function of frequency.
- Compute ρ at each frequency using Equation (9), obtained by rearranging Equation (5).

$$r = \frac{a}{pf} \tag{9}$$

- There will be some variation as a function of frequency, depending upon the coherence of the transfer function. Calculate the average ρ for the frequencies for which the transfer function can be assumed statistically adequate (and discard data "contaminated" by ambient conditions).
- Calculate the theoretical site-dependent material attenuation spectrum using Equation (10).

$$A_a = 8.68 r p f (r_b - r_a)\,. \tag{10}$$

- Add A_γ to A_α to obtain the attenuation spectrum.

These steps were carried out using the measured data from the scraper and vibratory compactor, with the resulting "fit" curves of $A_g + A_a$ shown in Figure 1. The values of α are between 0.002 and 0.02; the average value between 8 and 12 Hz is 0.007. The material damping values of the "fit" equations for scraper and compactors, respectively, are $\rho = 2.2 \times 10^{-5}$ and 2.1×10^{-5}.

The conclusion one may draw from this form of attenuation curve is that higher-frequency vibrations are attenuated more rapidly with distance than are low-frequency components. At distances quite close to an impact source (such as a pile driver) the peaks in the time history are quite sharp—indicative of high frequencies—but at a great distance the peaks are smooth and more undulating—showing that the high frequency components have indeed been attenuated.

If a two-channel spectrum analyzer is available, the transfer functions can be measured directly. In addition, the analyzer can be used in time-domain mode to obtain the travel time between the two measurement locations of a pulse generated at the driving location. This can be used to calculate site-specific Rayleigh wave velocity. It should be noted that the scraper was able to define a reasonably shaped attenuation curve because it generated a time history that was somewhat stationary and relatively free of impulses. Other sources which generated vibrations more impulsive in nature did not produce such consistent results.

5. EXAMPLE

As an example, let us review a project in which it was necessary to determine construction setback distances from a particularly sensitive building on a semiconductor R&D and production campus. The contractor proposed a suite of equipment to be used during construction, and wished to know how close each type of equipment could be used without exceeding the facility's vibration specification (taken here to be VC-D, 6 micrometers/sec). Some of the equipment examined in this study included:

- Light excavation equipment
- Heavy excavation equipment, such as scrapers, graders and soil grinders
- Hauling equipment, such as track loaders and dump trucks
- Compacting equipment, including sheepsfoot compactors as well as static and vibratory smooth-drum rollers

The site-specific attenuation curve was determined in the manner already discussed. Then, maximum-hold rms spectra were obtained for each proposed item of equipment while it operated in a normal manner. Care was taken to perform the measurements at a distance of 50 ft.

Much of the excavation and haulage equipment produced broadband spectra that were similar in shape and maximum amplitude. The spectra produced by smooth static rollers was similar to those for general excavation equipment. The sheepsfoot roller also produced a broadband spectrum, but a much more severe amplitude and with a somewhat different frequency distribution. The vibratory rollers produced vibrations at a single frequency, which varied by model.

Figure 2 shows how the maximum spectral amplitude for each equipment group varied with distance. It is clear that one cannot assume a single relationship of amplitude vs distance which will be applicable to all construction equipment at a site.

SUMMARY

Vibration analyses of advanced technology facilities typically must consider frequency as well as amplitude of vibration. A soil propagation model has been proposed which will allow the use of site-specific, measurable, frequency dependent attenuation characteristics. A method has been proposed which allows *in-situ* determination of those frequency-dependent properties.

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Table 1. Summary of theoretical geometric attenuation coefficients, based on wave type.

Source	Wave Type	Measurement Point	γ	dB/doubling
Point on Surface	Rayleigh	Surface	0.5	3
Point on Surface	Body	Surface	2	12
Point at Depth	Body	Surface	1	6
Point at Depth	Body	Depth	1	6

Table 2. Summary of published geometric attenuation coefficients, γ.

Investigator	Soil Type	Geometric Attenuation	
		γ	dB/doubling
Wiss[1]	Sands	1.0	6
	Clays	1.5	9
Brenner & Chittikuladilok[7]	Surface sands	1.5	9
	Sand fill over soft clays	0.8 - 1.0	5 - 6
Attewell & Farmer[8]	Various soils, generally firm	1.0	6
Nicholls, Johnson & Duvall[9]	Firm soils and rock	1.4 - 1.7	8.5 - 10
Martin[10]	Clay	1.4	8.5
	Silt	0.8	5
Amick & Ungar[11]	Clay	1.5	9

Table 3. Summary of published material attenuation coefficients, α.

Investigator	**Soil Type**	Material Coefficient, α, ft^{-1}
Forssblad[12]	Silty gravelly sand	0.04
Richart[13]	4-6 in concrete slab over compact granular fill	0.006
Woods[14]	Silty fine sand	0.08
Barkan[3]	Saturated fine grain sand	0.03
	Saturated fine grain sand in frozen state	0.018
	Saturated sand with laminae of peat and organic silt	0.012
	Clayey sand, clay with some sand, and silt above water level	0.012
	Marly chalk	0.03
	Loess and loessial soil	0.03
	Saturated clay with sand and silt	0.012-0.036
Dalmatov, et al.[15]	Sand and silts	0.008-0.11
Clough and Chameau[16]	Sand fill over Bay Mud	0.015-0.06
	Dune sand	0.008-0.02
Peng[17]	Soft Bangkok clay	0.079-0.134

Table 4. Summary of attenuation as a function of earth material type.

Class	Description of Material	Attenuation Coefficient, α (ft^{-1}) at 5 Hz	ρ
I	*Weak or soft soils* (soil penetrates easily); loessy soils, dry or partially saturated peat and muck, mud, loose beach sand and dune sand, recently plowed ground, soft spongy forest or jungle floor, organic soils, topsoil	0.003-0.01	$2x10^{-4}$ to $6x10^{-4}$
II	*Competent soils* (can dig with shovel): most sands, sandy clays, silty clays, gravel, silts, weathered rock	0001-0.003	$6x10^{-5}$ to $2x10^{-4}$
III	*Hard soils* (cannot dig with shovel, must use pick to break up): dense compacted sand, dry consolidated clay, consolidated glacial till, some exposed rock	0.0001-0.001	$6x10^{-6}$ to $6x10^{-5}$
IV	*Hard, competent rock* (difficult to break with hammer): bedrock, freshly exposed hard rock	<0.0001	$< 6x10^{-6}$

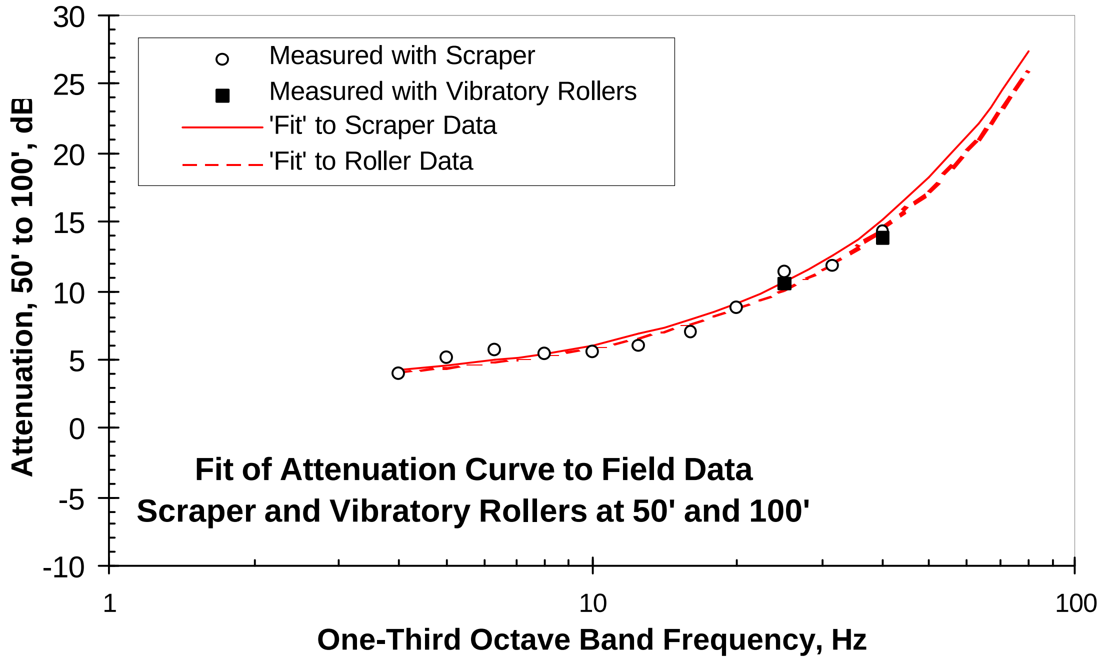

Figure 1. Measured and calculated material attenuation.

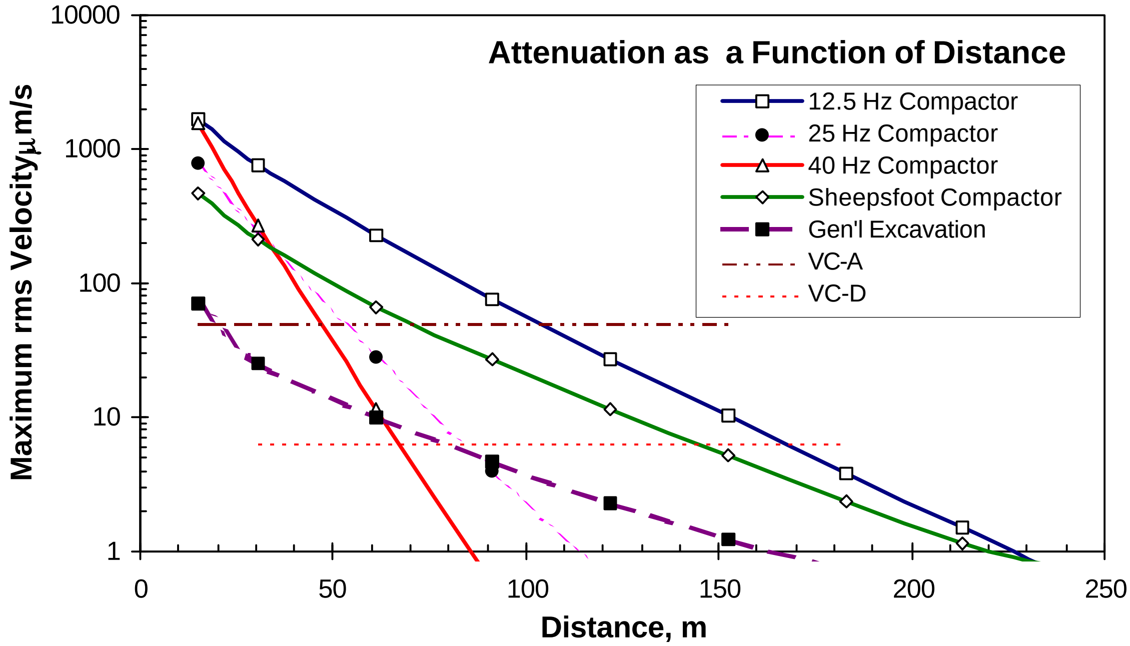

Figure 2. Example Site - Attenuation as a Function of Frequency Content and Distance