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ENCE 3610

Soil Mechanics

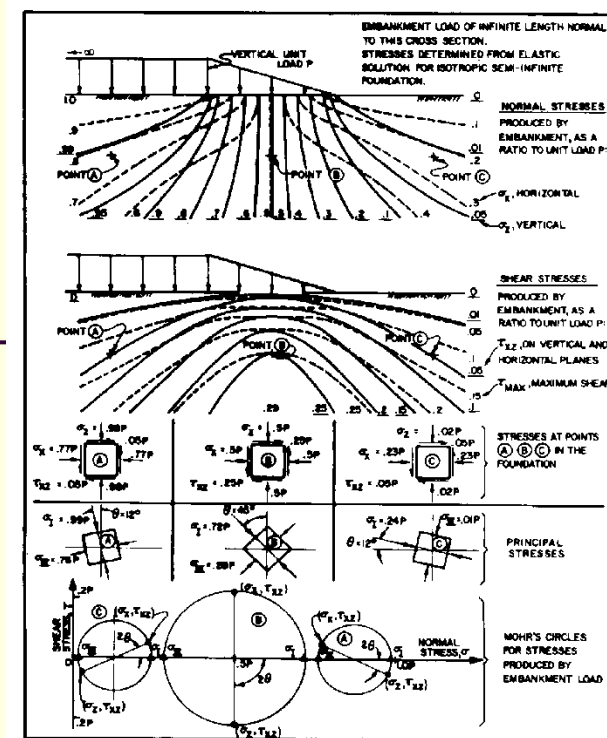


FIGURE 1
Examples of Stress Conditions at a Point

Lecture 9

Introduction to Soil Stress States

Mohr's Circle and Combined Stresses

Stresses in Soils

- Soils, like any other engineering material, experience stresses
 - We have already looked at stresses due to the “self-weight” of the soil and water combination (effective stresses)
- As structures are built around soils, the stress states of the soils will change
- Methods of state (stress and deflection) change
 - Elastic deformation and stress (non-path dependent,) similar to other engineering materials
 - Plastic deformation (path dependent,) which can be catastrophic under certain conditions
 - Consolidation settlement, volume change in soils due to rearrangement of the particles (unlike other engineering materials)

Stresses in Soils

- Soil stresses can and are analyzed with methods used elsewhere in mechanics of materials
- There are two important differences between soils and other materials and configurations which you may have studied
- Soils deform plastically almost exclusively
 - Our differentiation between elastic and plastic deformation and stress is, to a large extent, artificial, but necessary for analysis
- Soils act in a two- or three-dimensional continuum; for many students, it is the first introduction to continuum mechanics

Mohr's Circle in Two Dimensions

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left[\frac{\sigma_y - \sigma_x}{2}\right]^2 + \tau_{xy}^2}$$

$$\sigma_x = \frac{\sigma_1 + \sigma_3}{2} - \frac{(\sigma_1 - \sigma_3)\cos(2\theta)}{2}$$

$$\sigma_y = \frac{\sigma_1 + \sigma_3}{2} + \frac{(\sigma_1 - \sigma_3)\cos(2\theta)}{2}$$

$$\tau_{xy} = \tau_{yx} = \frac{(\sigma_1 - \sigma_3)\sin(2\theta)}{2}$$

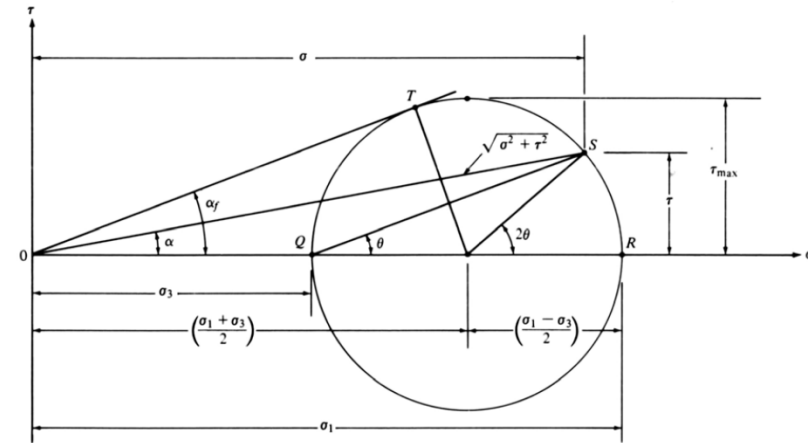


Figure B-2. Simplified version of Figure B-1(b) showing some basic relationships and characteristics of the Mohr's Circle (Cernica, 1982).

Table B-1

Summary of stresses acting on the four planes shown in Figure B-1(a)

Plane	Normal Stress	Sign	Shear Stress	Sign
P	σ_x	+	τ_{xy}	+
Q	σ_y	+	τ_{yx}	-
R	σ_x	+	τ_{yx}	+
S	σ_y	+	τ_{xy}	-

Note: In this table and Figure B-1, the sign convention is according to that commonly used in solid mechanics, i.e., tensile (normal) stresses are considered positive and are plotted to the right of the origin. In soil mechanics it is customary to indicate *compressive* stresses as positive because soil cannot sustain tensile stresses since it has virtually no strength in tension. The use of the soil mechanics convention will be illustrated in Sections B.3 and B.4.

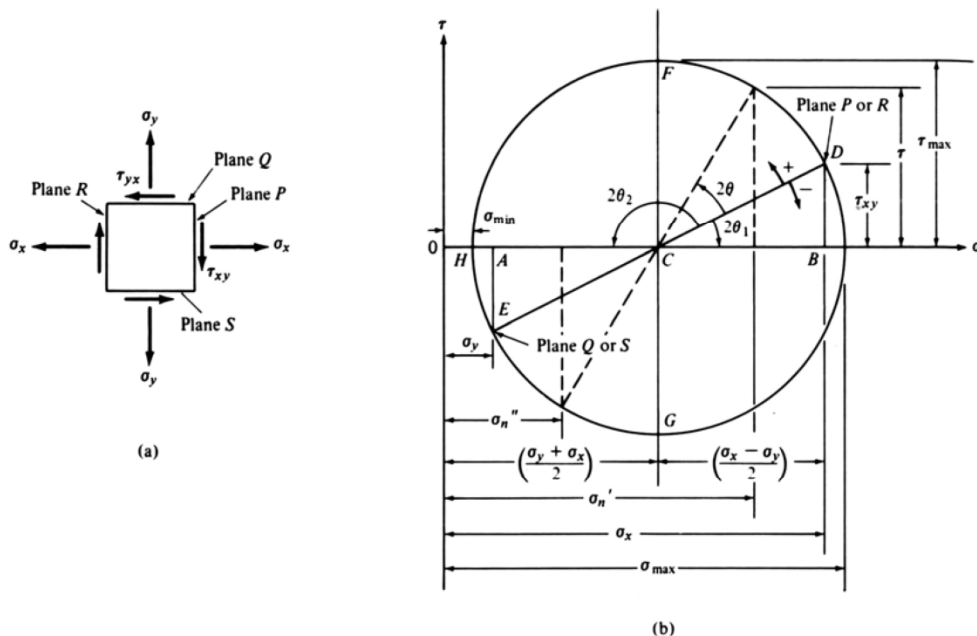
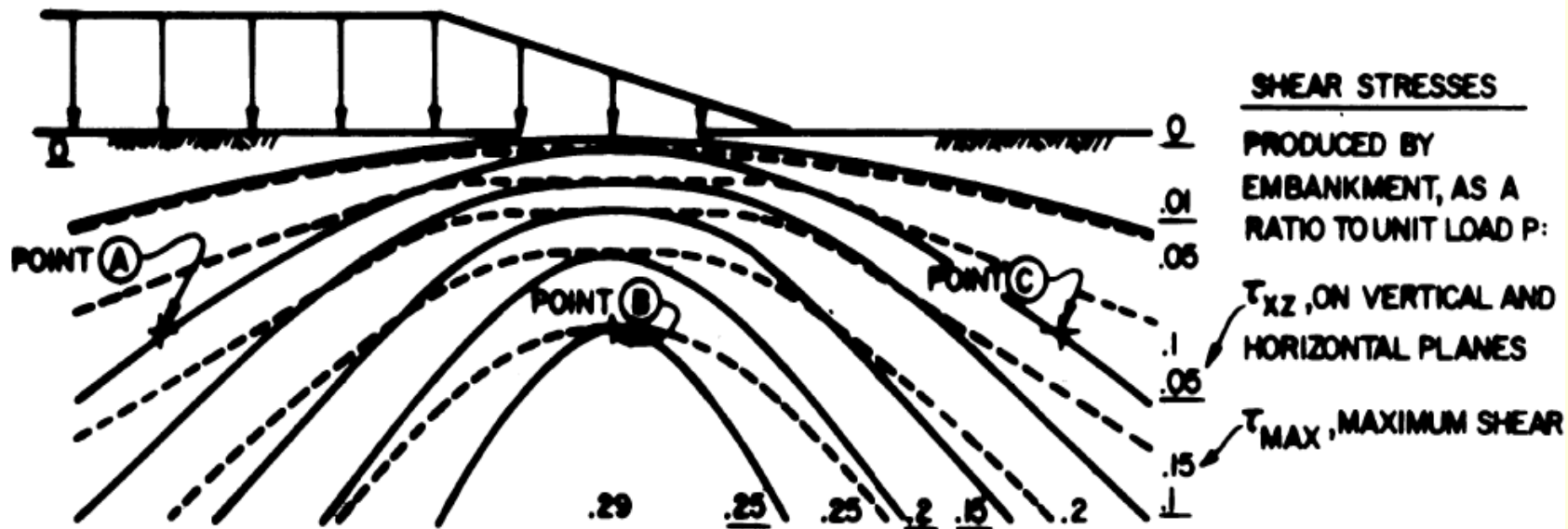
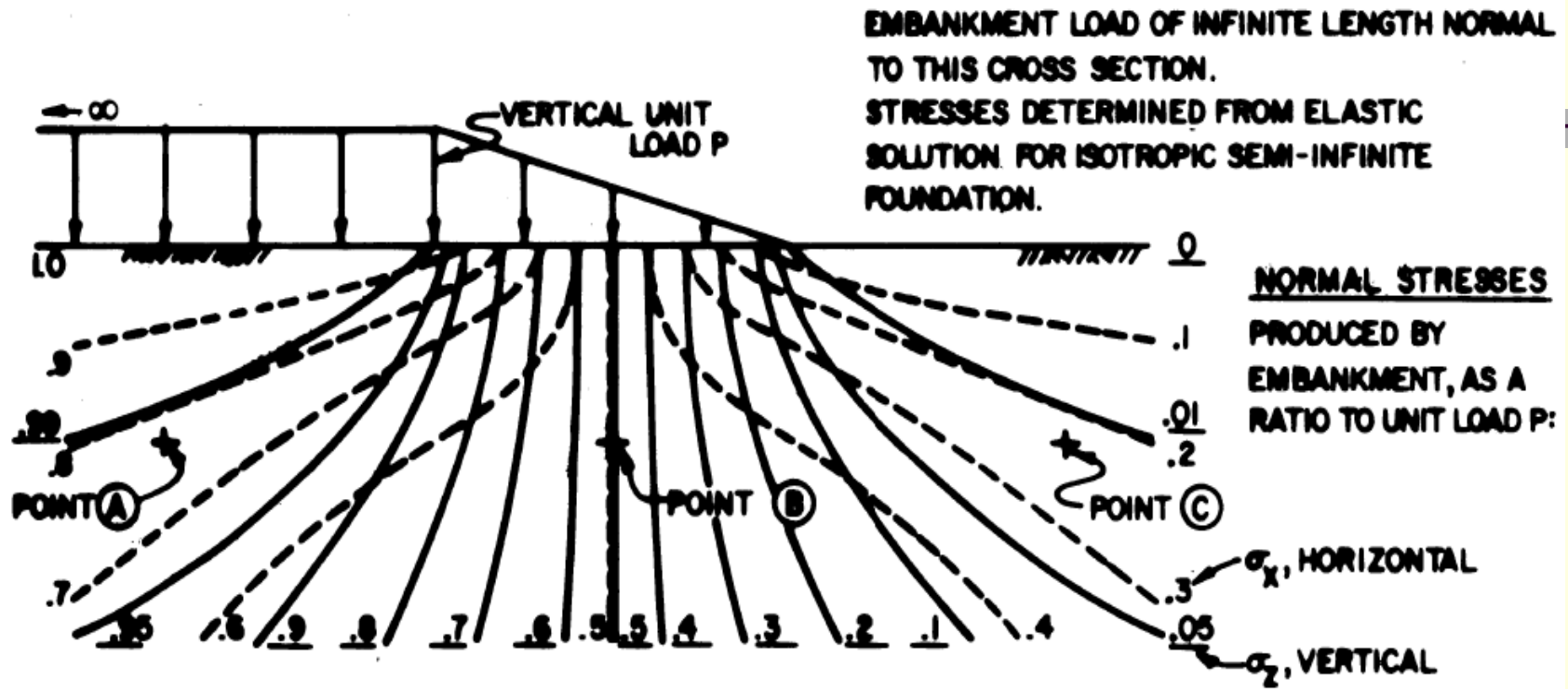
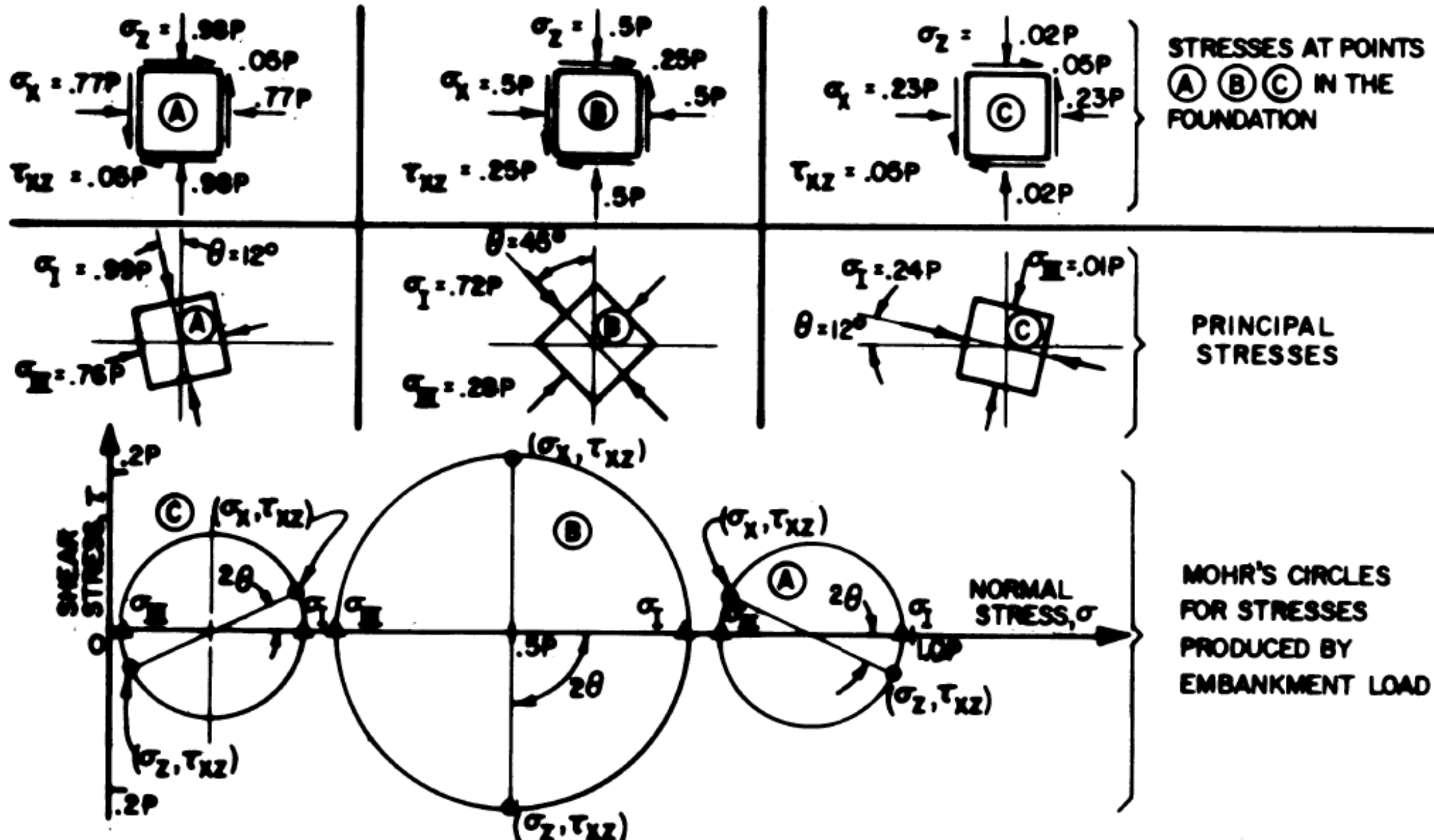


Figure B-1. Mohr's circle for general stress conditions (Cernica, 1982).

Examples Using Mohr's Circle



Examples Using Mohr's Circle



Observations on Principal and Shear Stresses in Soils

- Tension is never allowed in soils; compressive stresses are considered positive
 - Although principal stresses in many cases coincide with the z-y plane used in geotechnical analysis, this is not always the case
 - The understanding of principal and shear stresses is critical in determining the bearing and frictional capacities of soils
- Stress Invariants
 - The sum of the principal stresses in any stress condition is always equal to the sum of the normal (non-shear) stresses
 - Thus, for two-dimensional Mohr's Circle using Mohr-Coulomb failure,
$$\sigma_1 + \sigma_3 = \sigma_x + \sigma_y$$

Frictional Forces

$$F_a = W \sin \theta_m$$

$$P_n = W \cos \theta_m$$

$$\frac{F_a}{P_n} = \tan \theta_m$$

By analogy:

$$\frac{\tau}{\sigma} = \tan \theta \text{ or } \tau = \sigma \tan \theta$$

More completely:

$$\tau_{crit} = c + \sigma' \tan \theta$$

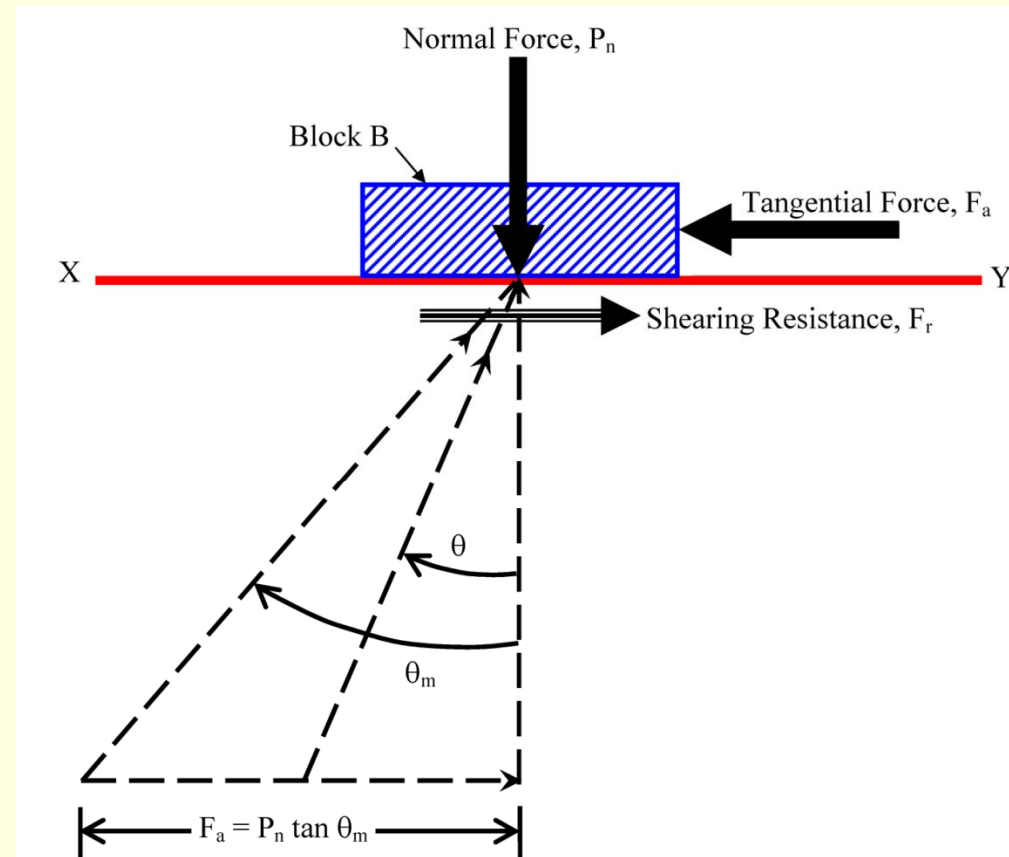
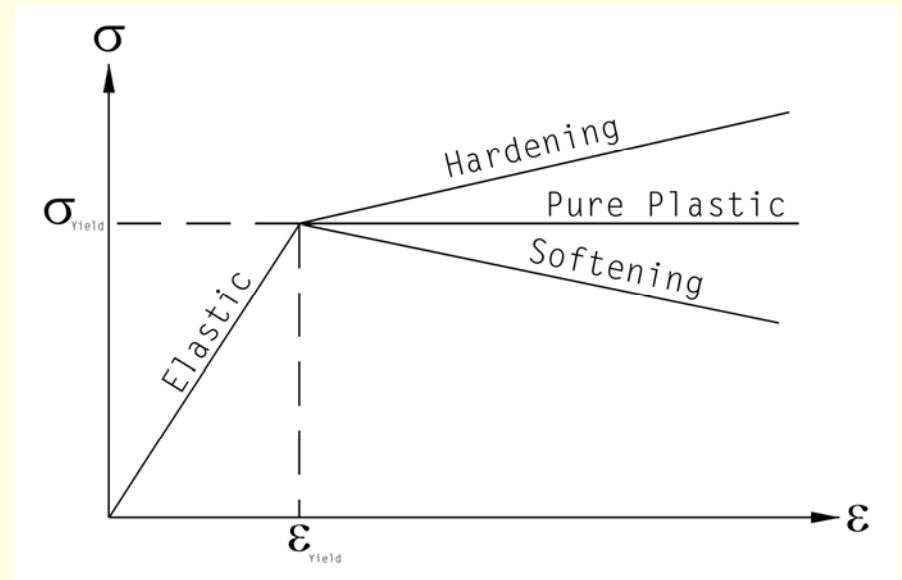


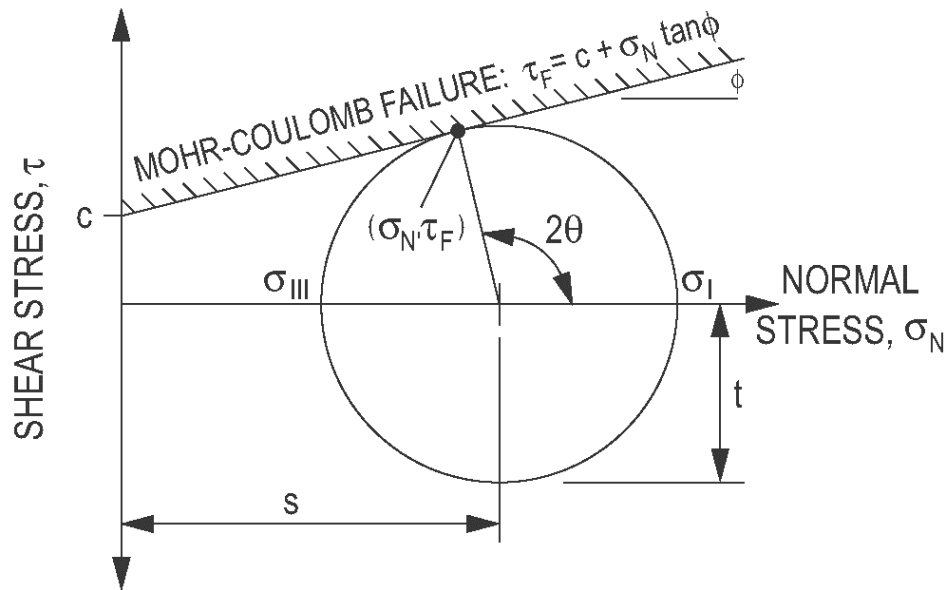
Figure 2-17. Basic concept of shearing resistance and strength (after Murthy, 1989).

Elastic-Purely Plastic Deformation

- Suggested by the frictional model
 - The block friction is able to resist movement up to a point, after which the movement is continuous with the applied force
- Not a perfect model, but one which is adequate for much analysis in soil mechanics



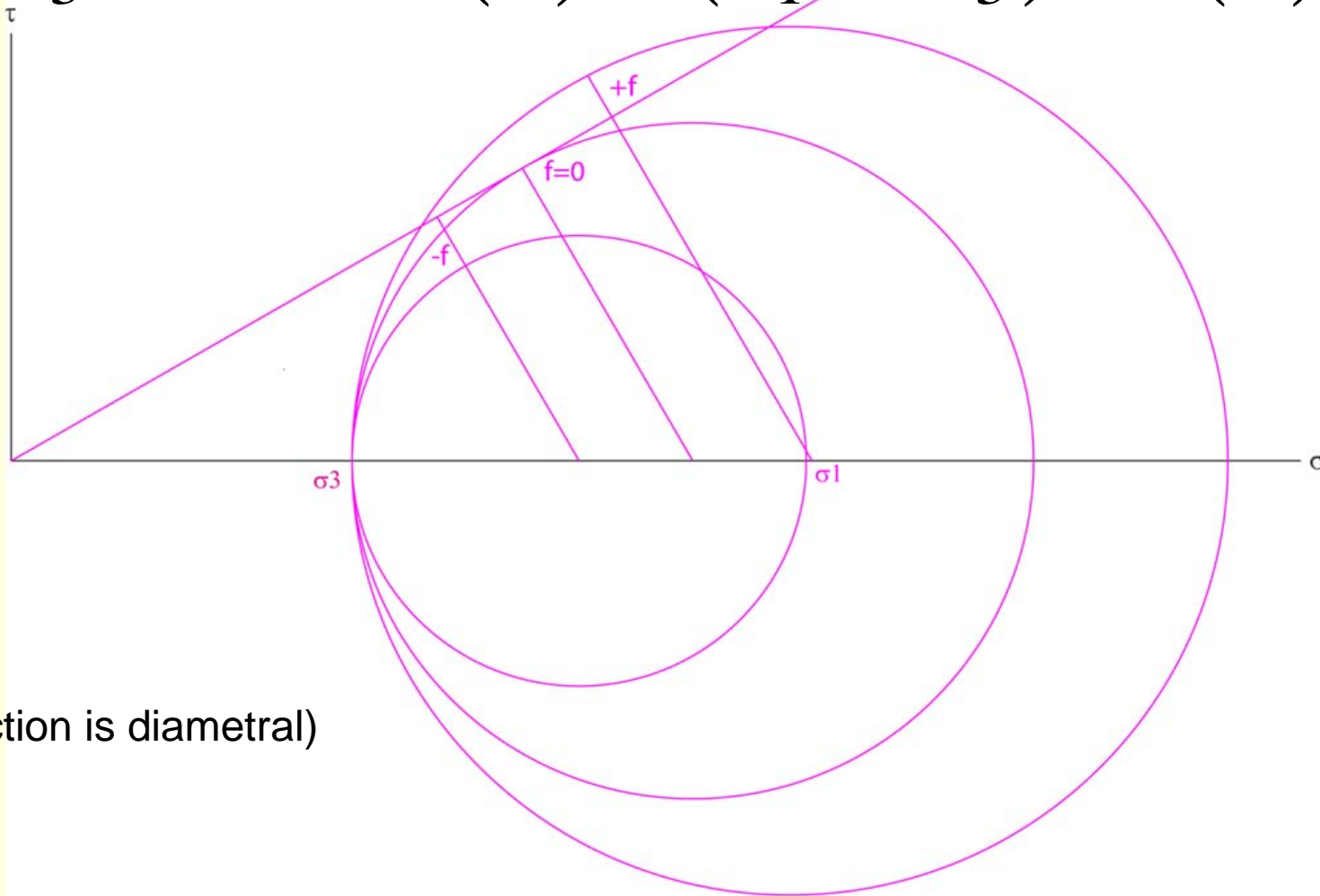
Failure Criteria of Soils



- s = mean normal stress
- t = maximum shear stress
- σ_I = major principal stress
- σ_{III} = minor principal stress
- θ = orientation angle between plane of existing normal stress and plane of major principal stress
- τ_F = shear stress at failure
- $\tau_F = c + \sigma_N \tan \phi$, by Mohr-Coulomb criterion
- c = cohesion
- ϕ = angle of internal friction
- q_U = unconfined compressive strength = $2c$ in unconfined compression test
- K_F = σ_{III}/σ_I (at failure)

Illustration of Failure and Non-Failure

$$\sigma_1 - \sigma_3 - 2c \cos(\phi) - (\sigma_1 + \sigma_3) \sin(\phi) = f$$



(Failure function is diametral)

$$f = -c \cos(\phi) - 1/2 \sin(\phi) \sigma_1 - 1/2 \sin(\phi) \sigma_3 + 1/2 \sigma_1 - 1/2 \sigma_3$$

Mohr-Coulomb Failure Theory

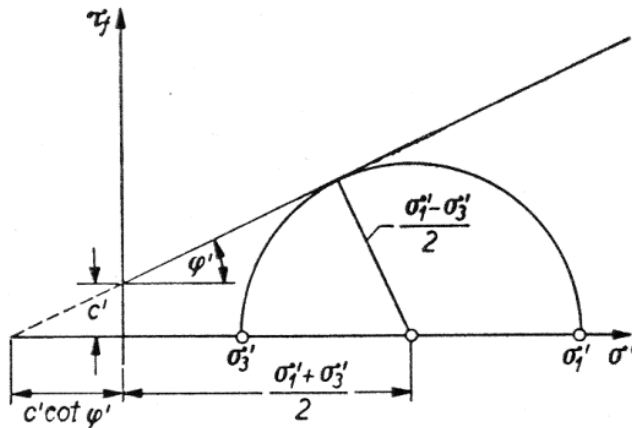


Fig. 6.3. Notation to Equation (6.6)

Using the notation of Fig. 6.3 the Coulomb law of shearing resistance (6.2) can be expressed, when interpreted according to Mohr's failure theory, as follows:

$$\frac{\sigma'_1 - \sigma'_3}{2} = \left(c' \cot \varphi' + \frac{\sigma'_1 + \sigma'_3}{2} \right) \sin \varphi'$$

$$\sigma'_1 - \sigma'_3 = 2c' \cos \varphi' + (\sigma'_1 + \sigma'_3) \sin \varphi' \quad (6.6)$$

In the case $c' = 0$, Equation (6.6) yields:

$$\frac{\sigma'_3}{\sigma'_1} = \frac{1 - \sin \varphi'}{1 + \sin \varphi'} = \tan^2 \left(\frac{\pi}{4} - \frac{\varphi'}{2} \right) \quad (6.7)$$

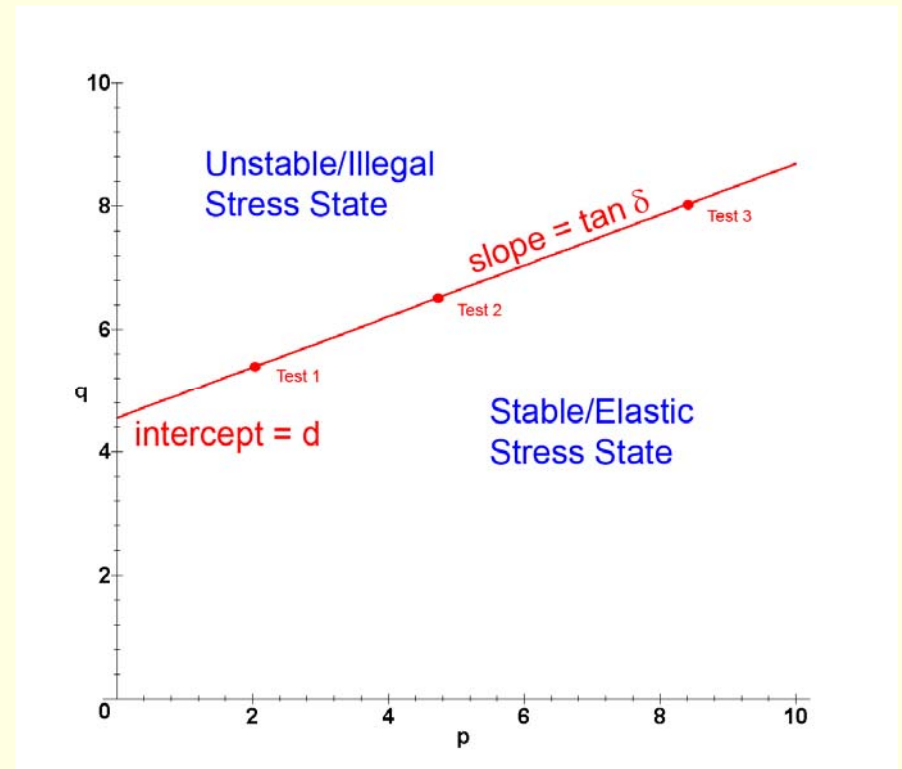
$$\sin \phi = \frac{\frac{1}{2}(\sigma'_1 - \sigma'_3)}{c \cot \phi + \frac{1}{2}(\sigma'_1 + \sigma'_3)}.$$

$$\sigma'_1 = \sigma'_3 \frac{1 + \sin \phi}{1 - \sin \phi} + 2c \frac{\cos \phi}{1 - \sin \phi}.$$

$$\sigma'_3 = \sigma'_1 \frac{1 - \sin \phi}{1 + \sin \phi} - 2c \frac{\cos \phi}{1 + \sin \phi}.$$

p-q Diagram

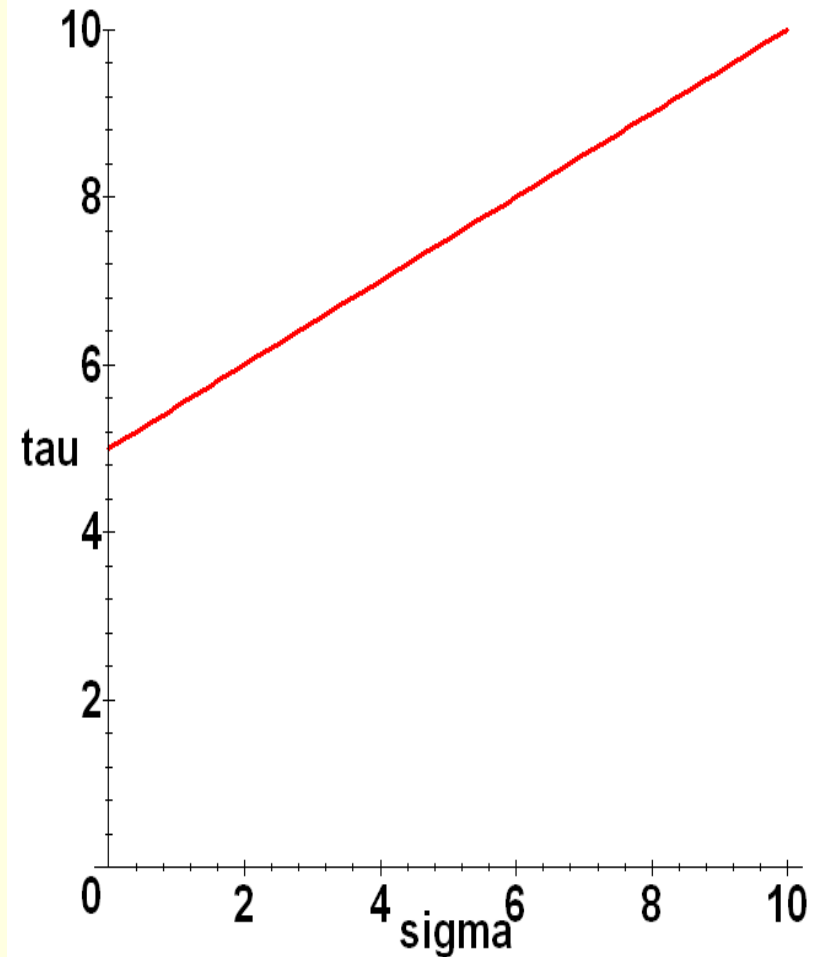
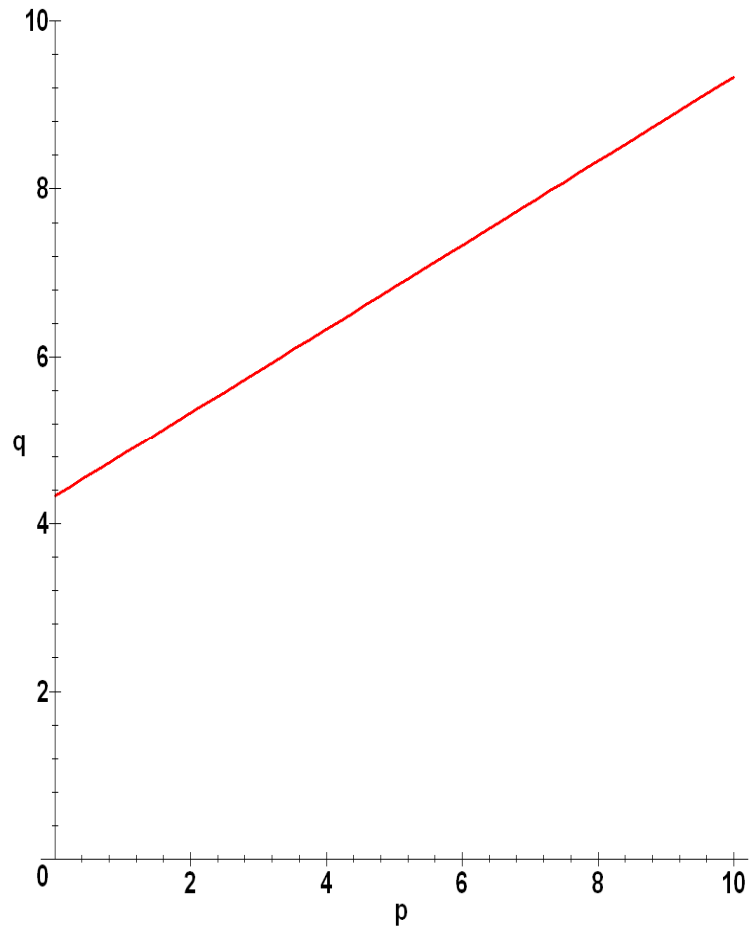
- A different method of expressing stress states than Mohr's Circle
- Makes analysis simpler in many cases
- Material on p-q diagrams elsewhere on site

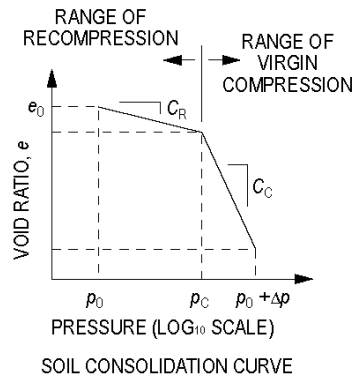


Example

- Given
 - Soil, $c = 5$ kPa, $\phi = 30$ deg.
- Find
 - Failure line plots for both sigma-tau and p-q diagrams
- Solution
 - The cohesion c is the same for both diagrams
- Solution
 - For tau-sigma diagram
$$\tau = c + \sigma \sin \phi$$
 - For p-q diagram
$$q = c\sqrt{1 - \tan^2 \delta} + p \tan \delta$$
 - Since $\tan \delta = \sin \phi$, the slope of both lines is the same, although the dependent variable (x-axis) is different
 - From substitution, $\delta = 26.6$ deg., and the y-intercept for the p-q diagram = 4.33 kPa

Example: Resulting Diagrams





e_0 = initial void ratio (prior to consolidation)
 Δe = change in void ratio
 p_0 = initial effective consolidation stress, σ'_0
 p_c = past maximum consolidation stress, σ'_c
 Δp = induced change in consolidation stress at center of consolidating stratum

$\Delta p = I q_s$
 I = Stress influence value at center of consolidating stratum
 q_s = applied surface stress causing consolidation

If $(p_0 \text{ and } p_0 + \Delta p) < p_c$, then $\Delta H = \frac{H_0}{1 + e_0} \left[C_R \log \frac{p_0 + \Delta p}{p_0} \right]$

If $(p_0 \text{ and } p_0 + \Delta p) > p_c$, then $\Delta H = \frac{H_0}{1 + e_0} \left[C_C \log \frac{p_0 + \Delta p}{p_c} \right]$

If $p_0 < p_c < (p_0 + \Delta p)$, then $\Delta H = \frac{H_0}{1 + e_0} \left[C_R \log \frac{p_c}{p_0} + C_C \log \frac{p_0 + \Delta p}{p_c} \right]$

where: ΔH = change in thickness of soil layer

Compression index

In virgin compression range: $C_C = \Delta e / \Delta \log p$

By correlation to liquid limit: $C_C = 0.009 (LL - 10)$

Recompression index

In recompression range: $C_R = \Delta e / \Delta \log p$

By correlation to compression index, C_C : $C_R = C_C / 6$

Ultimate consolidation settlement in soil layer

$$S_{ULT} = \epsilon_v H_s$$

H_s = thickness of soil layer

$$\epsilon_v = \Delta e_{TOT} / (1 + e_0)$$

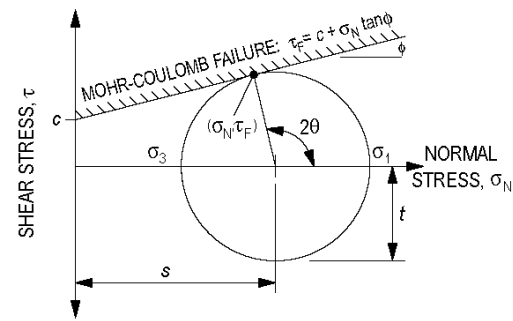
Δe_{TOT} = total change in void ratio due to recompression and virgin compression

Approximate settlement (at time $t = t_c$)

$$S_T = U_{AV} S_{ULT}$$

U_{AV} = average degree of consolidation

t_c = elapsed time since application of consolidation load



s = mean normal stress

t = maximum shear stress

σ_1 = major principal stress

σ_3 = minor principal stress

θ = orientation angle between plane of existing normal stress and plane of major principal stress

Total normal stress

$$\sigma_N = P/A$$

P = normal force

A = cross-sectional area over which force acts

Effective stress

$$\sigma' = \sigma - u$$

$u = h_u \gamma_w$

h_u = uplift or pressure head

Shear stress

$$\tau = T/A$$

T = shearing force

Shear stress at failure

$$\tau_F = c + \sigma_N \tan \phi$$

c = cohesion

ϕ = angle of internal friction

Questions?

