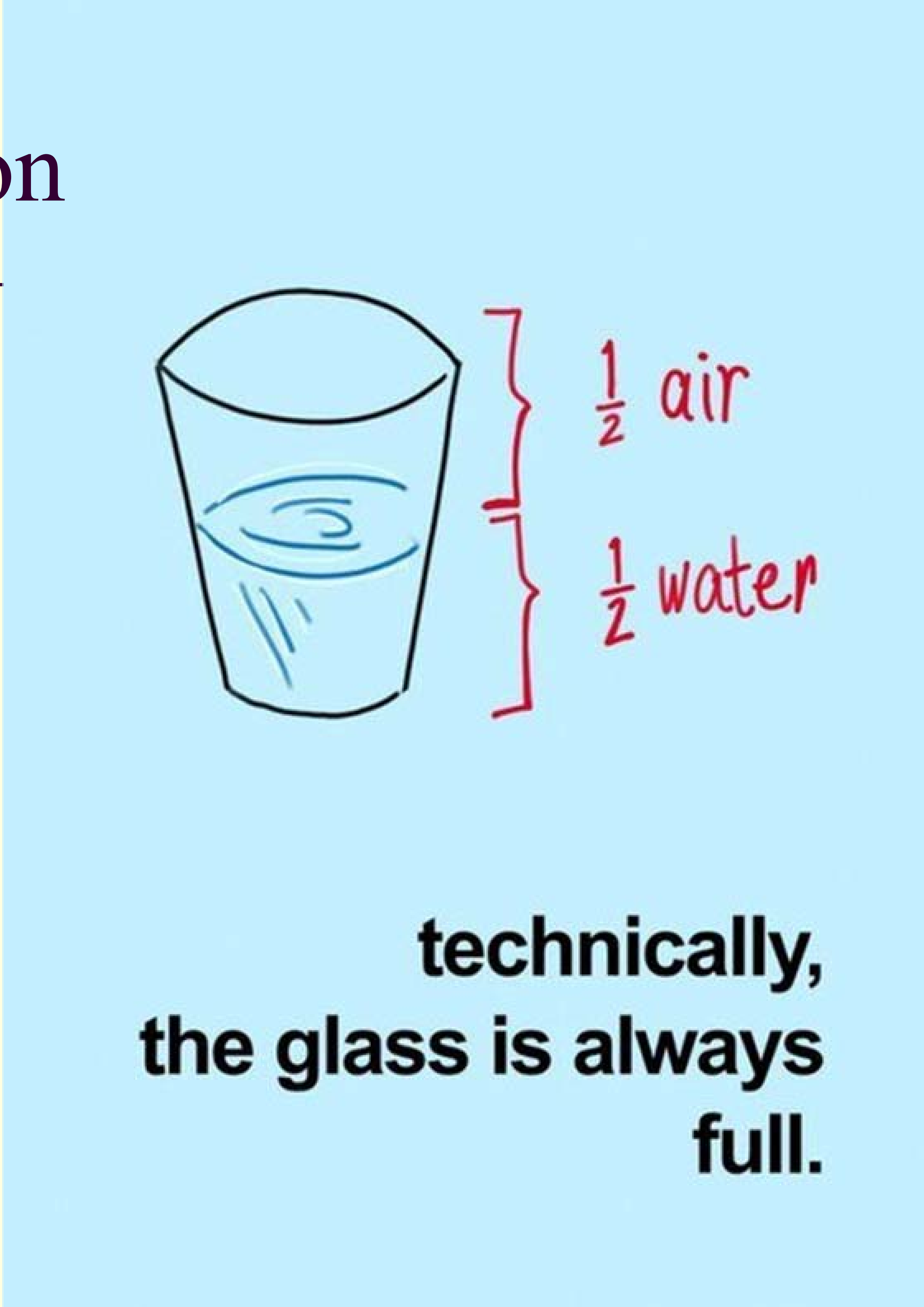
ENCE 361
Soil Mechanics

Lecture 4
Soil Composition

Soil Composition

- Weight-Volume Relationships
  - Water or Moisture Content
  - Unit Weight or Mass
  - Specific Gravity
- Relative Density
- Soil is a collection of particles that do not form an totally solid substance
- Soil is a combination of:
  - Soil material in particles
  - Air
  - Water
- The relationship between this combination defines much of what any particular soil can do to support foundations



Phase Diagram

- Assumptions and Definitions:
  - Weight of air = 0
  - Dry Soil: Water weight and volume = 0, all voids filled with air
  - "Volume of voids" include all non-soil volume, both air and water
- Equalities and Summations
  - $V_{total}=V_{air}+V_{water}+V_{solids}$
  - $W_{total}=W_{water}+W_{solids}$
  - $M_{total}=M_{water}+M_{solids}$
  - $W_x=\gamma_x*V_x$ or $M_x=\rho_x*V_x$

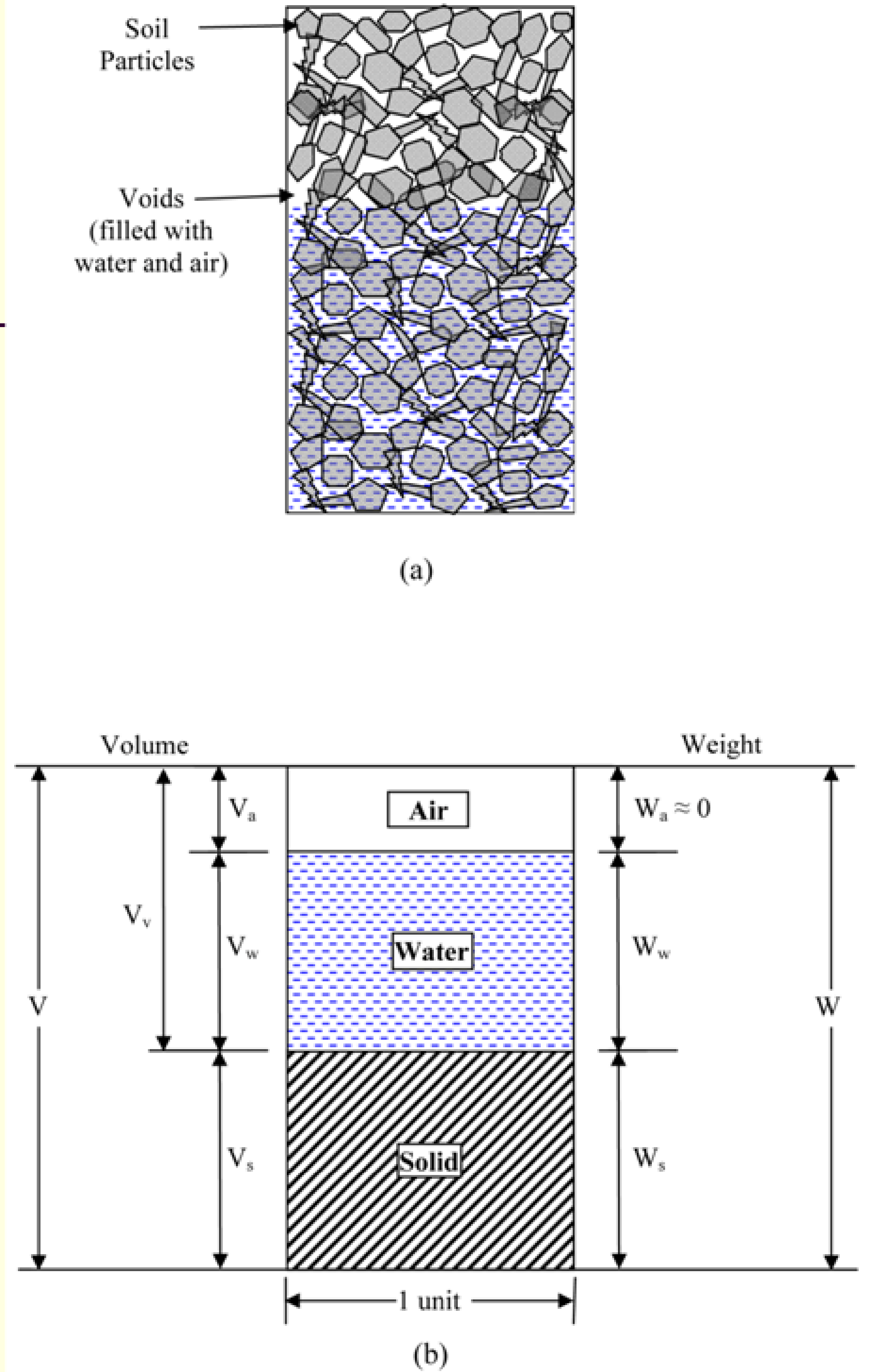


Figure 2-1. A unit of soil mass and its idealization.

Index Properties

An important basic parameter is the *porosity* n, defined as the ratio of the volume of the pore space and the total volume of the soil,

$$n = V_p/V_t. \qquad (3.1)$$

The amount of pores can also be expressed by the *void ratio* e, defined as the ratio of the volume of the pores to the volume of the solids,

$$e = V_p/V_s. \qquad (3.2)$$

The pores of a soil may contain water and air. To describe the ratio of these two the *degree of saturation* S is introduced as

$$S = V_w/V_p. \qquad (3.5)$$

The *water content* is another useful parameter, especially for clays. It has been used in the previous chapter. By definition the water content w is the ratio of the weight (or mass) of the water and the solids,

$$w = W_w/W_p. \qquad (3.12)$$

unit weight is reasonably constant. Using this logic, the **specific gravity of the soil solids**, G_s, can be expressed as follows:

$$G_S = \frac{\gamma_S}{\gamma_w} \qquad 2\text{-}10$$

The unit weight of a soil mass and the density/specific gravity of its particles is not the same!

Weight-Volume Relationships and Index Properties

Table 2-1

Summary of index properties and their application

Property	Symbol	Units[1]	How Obtained (AASHTO/ASTM)	Comments and Direct Applications
Porosity	n	Dim	From weight-volume relations	Defines relative volume of voids to total volume of soil
Void Ratio	e	Dim	From weight-volume relations	Volume change computations
Moisture Content	w	Dim	By measurement (T 265/ D 4959)	Classification and in weight-volume relations
Total unit weight [2]	γ_t	FL^{-3}	By measurement or from weight-volume relations	Classification and for pressure computations
Specific Gravity	G_s	Dim	By measurement (T 100/D 854)	Volume computations

NOTES:

1 F=Force or weight; L = Length; Dim = Dimensionless. Although by definition, moisture content is a dimensionless decimal (ratio of weight of water to weight of solids) and used as such in most geotechnical computations, it is commonly reported in percent by multiplying the decimal by 100.

2 Total unit weight for the same soil can vary from "saturated" (S=100%) to "dry" (S=0%).

Table 2-2

Weight-volume relations (after Das, 1990)

Unit-Weight Relationship	Dry Unit Weight (No Water)	Saturated Unit Weight (No Air)
$\gamma_t = \frac{(1+w)G_s\gamma_w}{1+e}$	$\gamma_d = \frac{\gamma_t}{1+w}$	$\gamma_{sat} = \frac{(G_s+e)\gamma_w}{1+e}$
$\gamma_t = \frac{(G_s+Se)\gamma_w}{1+e}$	$\gamma_d = \frac{G_s\gamma_w}{1+e}$	$\gamma_{sat} = [(1-n)G_s+n]\gamma_w$
$\gamma_t = \frac{(1+w)G_s\gamma_w}{1+\frac{wG_s}{S}}$	$\gamma_d = G_s\gamma_w(1-n)$	$\gamma_{sat} = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$
$\gamma_t = G_s\gamma_w(1-n)(1+w)$	$\gamma_d = \frac{G_s\gamma_w}{1+\frac{wG_s}{S}}$	$\gamma_{sat} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$
	$\gamma_d = \frac{eS\gamma_w}{(1+e)w}$	$\gamma_{sat} = \gamma_d + n\gamma_w$
	$\gamma_d = \gamma_{sat} - n\gamma_w$	$\gamma_{sat} = \gamma_d + \left(\frac{e}{1+e}\right)\gamma_w$
	$\gamma_d = \gamma_{sat} - \left(\frac{e}{1+e}\right)\gamma_w$	

In above relations, γ_w refers to the unit weight of water, 62.4 pcf (=9.81 kN/m^3).

Systems of Units Used in Phase Calculations

- Weight and Volume Relationships
  - In most cases, calculations in soil mechanics are done on a weight basis (SI or US units)
  - Exceptions include wave propagation problems (earthquakes, pile dynamics, etc.
- U.S. Units
  - Most common within the U.S., generally not used elsewhere
  - Basic units of weight: 1 pound (lb.) or kip (1000 lbs.)
  - Basic unit of length: feet
  - Will frequently write psf and pcf instead of lb/ft^2 and lb/ft^3
- SI Units
  - Technically the "official" system everywhere else
  - Basic units of weight: N or kN
  - Basic unit of length: m
- MKS Units
  - Commonly used in "metric" countries
  - Basic unit of mass: kg
  - Basic unit of length: m

Specific Gravity and Density

- Unit Weight of Water (γ_w)
  - 62.4 pcf = 0.624 kcf
  - 9.81 kN/m^3
- Density of Water
  - 1.95 slugs/ft^3
  - 1000 kg/m^3
  - 1 g/cm^3 = 1 Mg/m^3 = 1 Metric Ton/m^3
- Typical Values for Soil Particles
  - Quartz Sand: 2.64 – 2.66
  - Silt: 2.67 – 2.73
  - Clay: 2.70 – 2.9
  - Chalk: 2.60 – 2.75
  - Loess: 2.65 – 2.73
  - Peat: 1.30 – 1.9
  - Except for organic soils, range is fairly narrow

Example 1

- Given:
  - Total Volume = 1 cu. ft.
  - Total Weight = 140 lb.
  - Dry Weight = 125 lb.
- Find
  - Water Content
  - Wet Unit Weight
  - Dry Unit Weight

- By Definition:
  - Dry Unit Weight = Dry Weight = 125 lb/ft^3
  - Wet Unit Weight = Total Weight = 140 lb/ft^3
- Solve for Weight of Water
  - $W_T = W_s + W_w$
  - $140 = 125 + W_w$
  - $W_w = 15\ lb/ft^3$
- Solve for Water Content
  - $w = W_w/W_s = W_w/125 = 15/125 = 0.12 = 12\%$

Example 2

- Given:
  - Total Mass = 18.18 kg
  - Total Volume = 0.009 m^3
  - Dry Mass = 16.13 kg
  - Specific Gravity of Solids = 2.7
- Find
  - Wet Density
  - Dry Unit Weight
  - Void Ratio
  - Water Content
  - Degree of Saturation

- Convert masses to weights
  - W_t = (18.18)(9.8) = 178.2 N = 0.178 kN
  - W_s = (16.13)(9.8) = 158.1 N = 0.158 kN
- Compute Weight of Water
  - $W_t = W_s + W_w$
  - $.178 = .158 + W_w$
  - $W_w = .02$ kN
- Compute Water Content
  - $w = W_w/W_s$
  - w = .02/.158 = .127 = 12.7%

Example 2

Compute Volumes

Volume of Water

- $V_w = W_w / \gamma_w$
- $V_w = .02\ kN/9.8\ kN/m^3 = 0.00205\ m^3$

Volume of Solids

- $V_s = W_s / \gamma_s = w_s/(G_s\ \gamma_w)$
- $V_s = 0.158/((9.8)(2.7)) = 0.00597\ m^3$

Volume of Air

- $V_a = V_t – V_w – V_s$
- $V_a = 0.009\text{-}0.00205\text{-}0.00597 = .00098\ m^3$

Compute Wet Unit Weight

$\gamma_{wet} = W_T/V_T = 0.178/0.009 = 19.78\ kN/m^3$

Compute Dry Unit Weight

$\gamma_{dry} = W_s/V_T = 0.158/0.009 = 17.58\ kN/m^3$

Void Ratio

$e = V_v/V_s = (V_w+V_a)/V_s = (.00205+0.000976)/.00597 = .507$

Compute Degree of Saturation

$S = V_w/(V_w+V_a)$

$S = .00205/(.000976+.00205) = .677 = 67.7\%$

Example 3

- Given
  - Saturated Soil
  - Void Ratio = 0.45
  - Specific Gravity of Solids = 2.65
- Find
  - Wet Unit Weight
  - Water Content
- Assumptions
  - $V_a = 0$
  - $V_t = 1$
  - $V_s + V_w = 1$
  - $\gamma_{water} = 62.4\ lb/ft^3$
- Solve for Volumes
  - $e = V_w/V_s = 0.45$
  - $V_w = 0.31\ ft^3$
  - $V_s = 0.69\ ft^3$
- Compute Wet Unit Weight
  - Weight of Solids = $\gamma_w V_s G_s$ = (62.4)(0.69)(2.7) = 114 lb
  - Weight of Water = $\gamma_w V_w$ = (62.4)(0.31) = 19.4 lb
  - Total Weight = 114 + 19.4 = 133.4 lb
  - Since volume is unity, total weight is also net unit weight = 133.4 pcf
- Compute Water Content
  - $w = W_w/W_s$ = 19.4/114 = 0.17 = 17%

Example 4

- Given
  - Well Graded Sand
  - Specific Gravity of Solids = 2.65
  - Void Ratio = 0.57
  - Porosity = 36.5%
  - Saturated Soil
- Find
  - Wet and Dry Unit Weight of Soil
- Solution
  - Set sample volume = 1 ft^3
  - Total Volume = 1= $V_w + V_a + V_s$
  - Use porosity to compute volume of voids
    - $n = V_v/V_t$
    - $V_v = nV_t = (.365)(1) = .365\ ft^3$
- Compute volume of solids
  - $V_s = V_t – V_v = 1 - .365 = .635\ ft^3$
- $V_v = V_w + V_a$
- Since soil is saturated, $V_a = 0$ and $V_w = .365\ ft^3$
- Dry Unit Weight
  - $Ws = \gamma_w G_s V_s =$ (62.4)(2.65)(.635) = 105 lb/ft^3
- Weight of Water
  - $W_w = \gamma_w V_w$ = (62.4)(.362) = 22.6 lb/ft^3
- Wet Unit Weight
  - $W_t = W_w + W_v$ = 127.6 lb/ft^3

Relative Density

- Density
  - Both unit weight and strength of soil can vary with particle arrangement
  - Denser soils have both higher load carrying capacity and lower settlement

Definition of Relative Density

$$RD = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

- e_{max} = void ratio of the soil in its loosest condition
- e_{min} = void ratio of the soil in its densest condition
- e = void ratio in the natural or condition of interest of the soil
- Convenient measure for the strength of a cohesionless soil

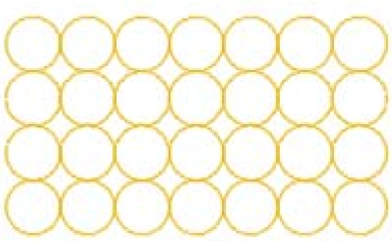


Figure 3.1: Cubic array.

$n = 0.4764$

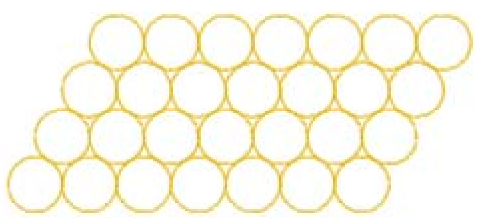


Figure 3.2: Densest array.

$n = 1 - \pi/\sqrt{18} = 0.2595$

Relative Density Example

- Given
  - Sand Backfill
  - Unit Weight = 109 pcf
  - Water Content = 8.6%
  - Specific Gravity of Solids = 2.6
  - e_{max} = 0.642 (loosest state)
  - e_{min} = 0.462 (densest state)
- Find
  - Void Ratio
  - Relative Density

- Assume $V_t = 1$ ft^3; thus, $W_t = 109$ lbs.
- Weight balance: $109 = W_s + W_w$
- Water content $w = W_w/W_s = 0.086$
- Solving two previous equations:
  - $W_s = 100.4$ lbs; $W_w = 8.6$ lbs.
  - $V_s = W_s/\gamma_s = 100.4/((2.6)(62.4)) = 0.618$ ft^3
  - $V_w = W_w/\gamma_w = 8.6/62.4 = 0.138$ ft^3
  - $V_a = V_t - V_w - V_s = 1 - 0.138 - 0.618 = 0.243$ ft^3
- $e = (V_a + V_w)/V_s = (0.243 + 0.138)/0.618 = 0.616$

$$RD = \frac{e_{max} - e}{e_{max} - e_{min}}$$

- RD = (.642-.616)/(.642-.462) = 0.144 = 14.4%

Use of Relative Density for Empirical Values

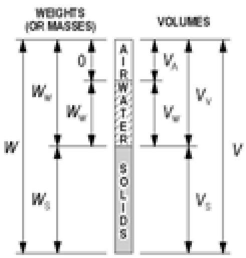
TABLE 4-6 EMPIRICAL VALUES FOR φ, D_r, AND UNIT WEIGHT OF GRANULAR SOILS BASED ON CORRECTED N' (after Bowles, 1977)

Description	Very Loose	Loose	Medium	Dense	Very Dense
Relative density D_r	0 - 0.15	0.15 - 0.35	0.35 - 0.65	0.65 - 0.85	0.85 - 1.00
Corrected Standard Penetration N' value	0 to 4	4 to 10	10 to 30	30 to 50	50+
Approximate angle of internal friction φ *	25 - 30°	27 - 32°	30 – 35°	35 - 40°	38 - 43°
Range of approximate moist unit weight, γ kN/m^3 (lb/ft^3)	11.0 - 15.7 (70 - 100)	14.1 - 18.1 (90 - 115)	17.3 - 20.4 (110 - 130)	17.3 - 22.0 (110 - 140)	20.4 - 23.6 (130 - 150)

Phase Relationships, Relative Density and Atterberg Limits

GEOTECHNICAL

Phase Relationships



PHASE DIAGRAM

Volume of voids
$V_V = V_A + V_W$

Total unit weight
$\gamma = W/V$

Saturated unit weight
$\gamma_{sat} = (G_s + e)\,\gamma_w/(1 + e) = \gamma(G_s + e)/(1 + \omega)$
$\gamma_W = 62.4 \text{ lb/ft}^3$ or 9.81 kN/m^3

Effective (submerged) unit weight
$\gamma' = \gamma_{sat} - \gamma_W$

Unit weight of solids
$\gamma_S = W_S/V_S$

Dry unit weight
$\gamma_D = W_S/V$

Water content (%)
$\omega = (W_W/W_S) \times 100$

Specific gravity of soil solids
$G_S = (W_S/V_S)/\gamma_W$

Void ratio
$e = V_V/V_S$

Porosity
$n = V_V/V = e/(1+e)$

Degree of saturation (%)
$S = (V_W/V_V) \times 100$

Relative density
$D_r = [(e_{max} - e)/(e_{max} - e_{min})] \times 100$
$= [(\gamma_{D\,field} - \gamma_{D\,min})/(\gamma_{D\,max} - \gamma_{D\,min})][\gamma_{D\,max}/\gamma_{D\,field}] \times 100$

Relative compaction (%)
$RC = (\gamma_{D\,field}/\gamma_{D\,max}) \times 100$

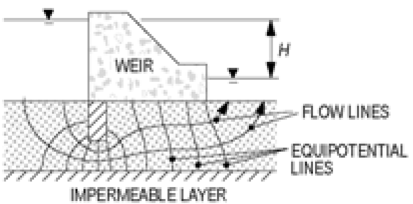
Plasticity index
PI = LL – PL
LL= liquid limit
PL = Plastic limit

Coefficient of uniformity
$C_U = D_{60}/D_{10}$

Coefficient of concavity (or curvature)
$C_C = (D_{30})^2/(D_{10} \times D_{60})$

Hydraulic conductivity (also coefficient of permeability)
From constant head test: $k = Q/(iAt_e)$
$i = dh/dL$
Q = total quantity of water
From falling head test: $k = 2.303[(aL)/(At_e)]\log_{10}(h_1/h_2)$
A = cross-sectional area of test specimen perpendicular to flow
a = cross-sectional area of reservoir tube
t_e = elapsed time
h_1 = head at time $t = 0$
h_2 = head at time $t = t_e$
L = length of soil column
Discharge velocity, $v = ki$

Flow Nets



FLOW NET

$Q = kH\,(N_f/N_d)$
N_f = number of flow channels
N_d = number of equipotential drops
H = total hydraulic head differential

Factor of safety against seepage liquifaction
$FS_s = i_c/i_e$
$i_c = (\gamma_{sat} - \gamma_W)/\gamma_W$
i_e = seepage exit gradient

Questions?