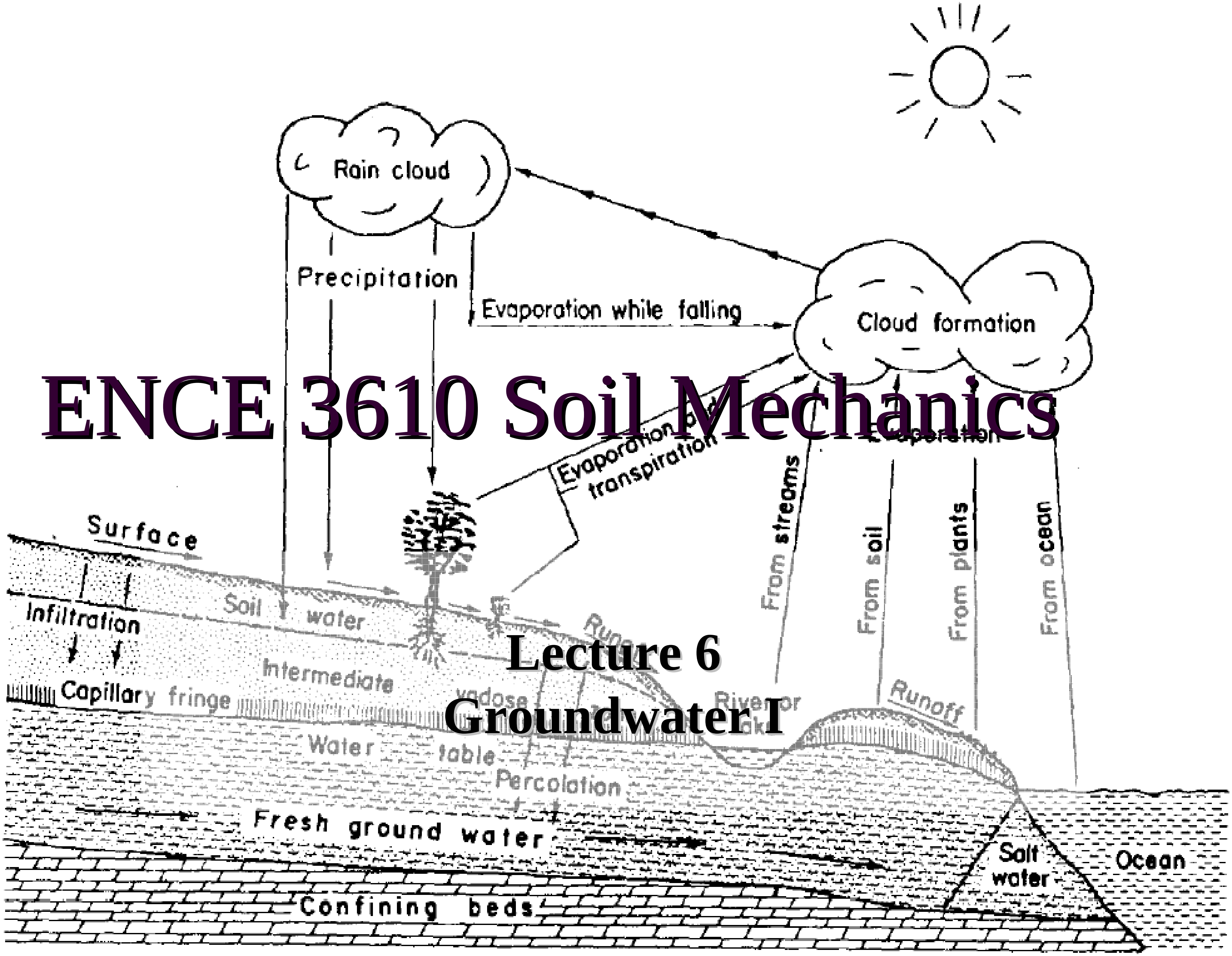
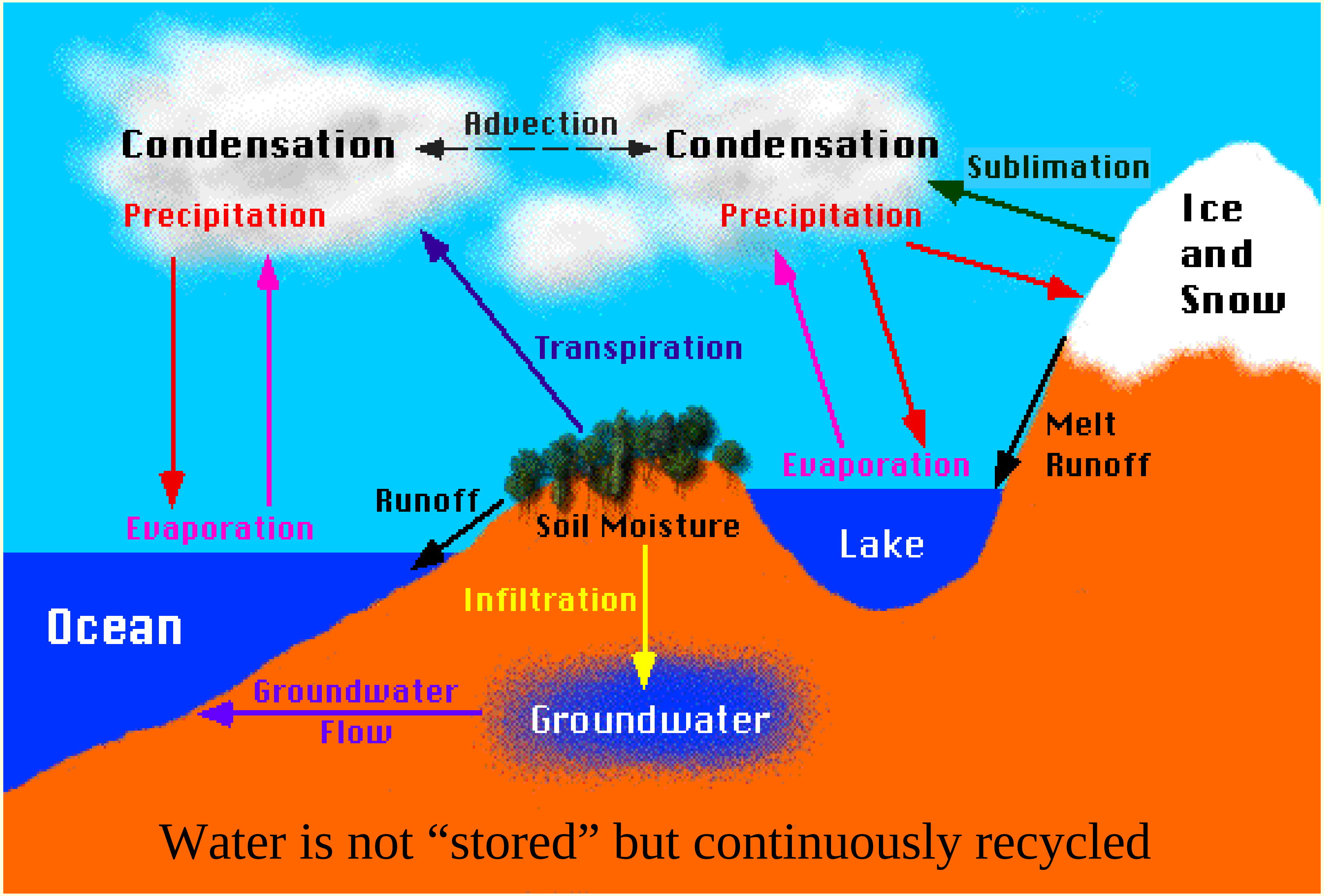


ENCE 3610 Soil Mechanics

Lecture 6
Groundwater I

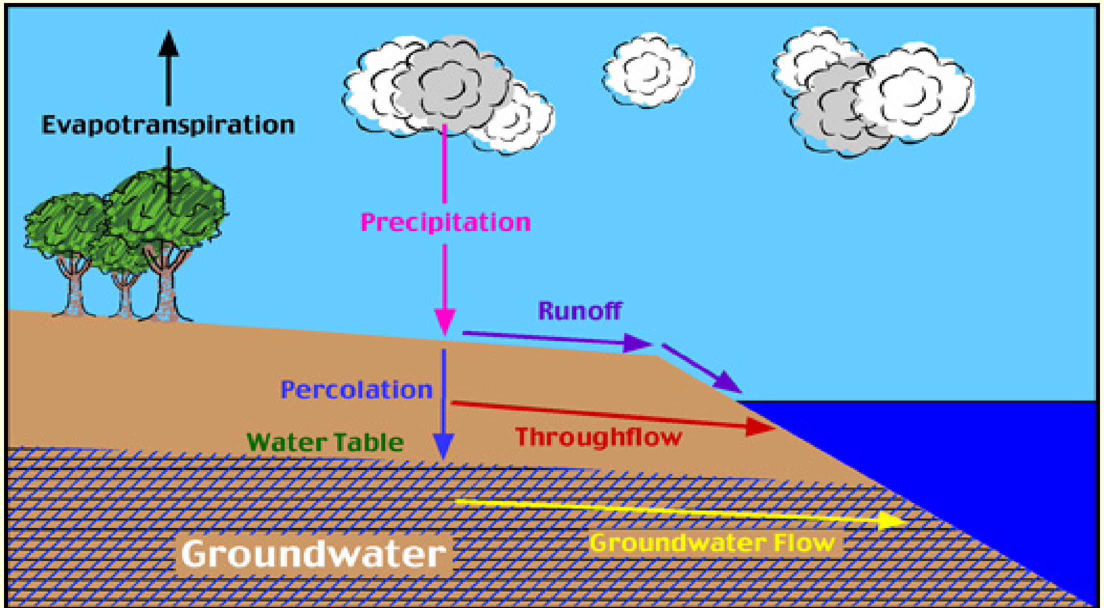
Hydrologic Cycle



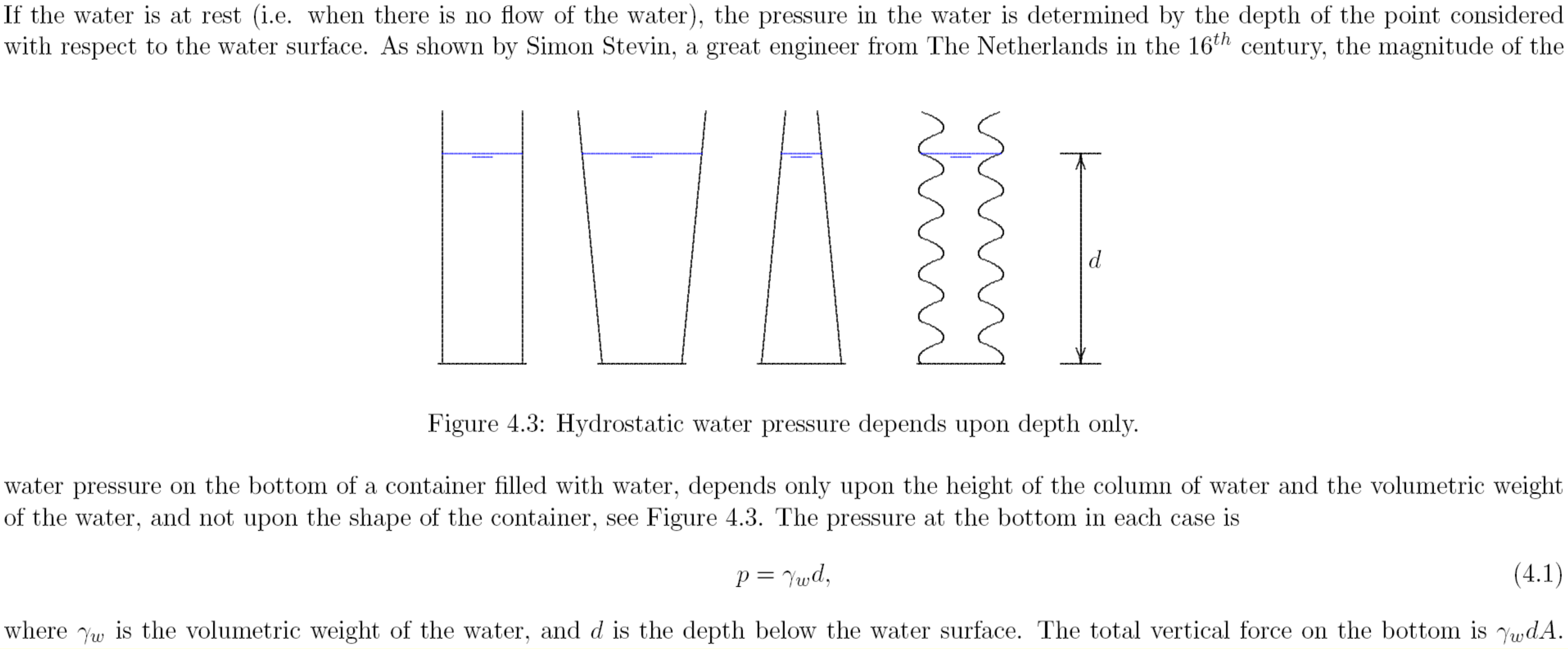
Water is not “stored” but continuously recycled

Aspects of Hydrology

- A relatively small amount of the earth's water (<1%) is contained in the groundwater, but the effects of this water are out of proportion to their amount
- The permeability of soil affects the distribution of water both between the surface and the ground mass and within the ground mass itself



Hydrostatic Condition

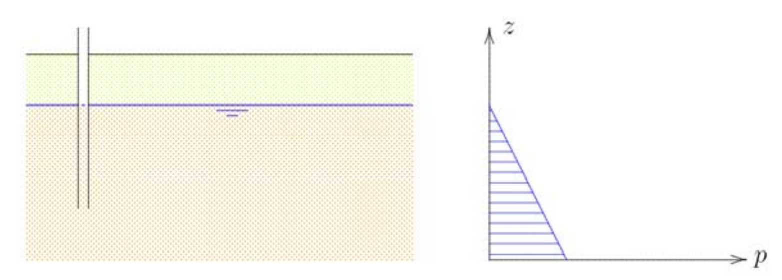


If the water is at rest (i.e. when there is no flow of the water), the pressure in the water is determined by the depth of the point considered with respect to the water surface. As shown by Simon Stevin, a great engineer from The Netherlands in the 16^{th} century, the magnitude of the water pressure on the bottom of a container filled with water, depends only upon the height of the column of water and the volumetric weight of the water, and not upon the shape of the container, see Figure 4.3. The pressure at the bottom in each case is

Figure 4.3: Hydrostatic water pressure depends upon depth only.

$$p = \gamma_w d, \tag{4.1}$$

where γ_w is the volumetric weight of the water, and d is the depth below the water surface. The total vertical force on the bottom is $\gamma_w dA$.



Changes in Groundwater Level

- Seasonal variations; variations due to drought, heavy rains, etc.
- Addition of artificial structures and elements to landscape
  - Reservoirs, dams, and artificial river channels
  - Construction site excavations
- Drawdown due to well water, drains, etc.

Permeability

- Definition
  - The property of soils that allows water to pass through them at some rate
  - This property is a product of the granular nature of the soil, although it can be affected by other factors (such as water bonding in clays)
  - Different soils have different permeabilities, understanding of which is critical to the use of the soil as a foundation or structural element

Theory for Moving Water

- Bernoulli's Law

$$h = \frac{v^2}{2g} + \frac{p}{\gamma} + z$$

  - Kinetic Component
  - Pressure Component
  - Gravitational (Potential) Component
- In reality, an energy balance of the soil as it flows through the ground
- Kinetic component can usually be ignored
  - Example: a 1 ft/sec velocity only produces 0.015 ft head difference for water
- We will ignore losses in the soil

- Variables
  - h=head at a given point
  - v=specific discharge
    - Sometimes referred to as the seepage velocity, but this is in reality different
  - g=acceleration due to gravity
  - p=pressure
  - γ =unit weight of fluid
  - z=position in gravity field

Specific Discharge

- The specific discharge does not refer to the actual velocity going through the pores of the soil
- It refers to an average velocity of a fluid flow though a given area, basically ignoring the presence of the pores
  - q=flow through a cross-sectional area of soil
  - A=cross-sectional area of soil

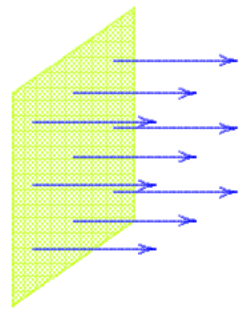


Figure 6.4: Specific discharge.

$$v = \frac{q}{A}$$

Hydraulic Gradient

- Equation:

$$i = \frac{\Delta h}{\Delta l}$$

  - i=hydraulic gradient (dimensionless)
  - Δ h=change in head from one point to another
  - Δ l=change in position from one point to another (distance)
- Equation presented in differential form (see Verruijt text)
  - This indicates that hydraulic gradient can and does vary continuously in the soil

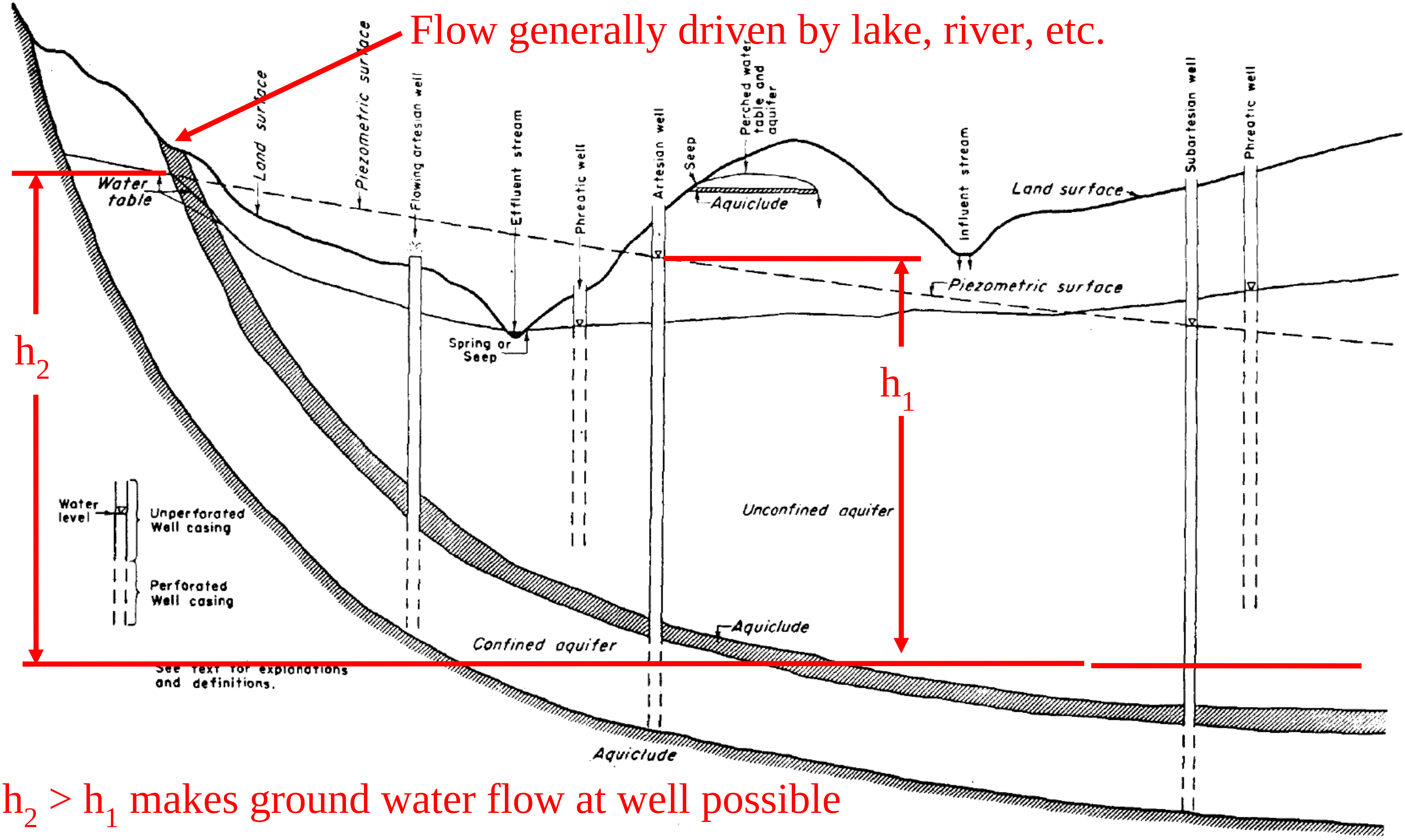
- Head at any point:

$$h = \frac{p}{\gamma} + z$$

- For purely horizontal flow, z is constant and the hydraulic gradient is expressed perfectly as shown on the left
- For vertical flow, gravity affects the hydraulic gradient and this needs to be taken into consideration during vertical flow

$$h_1 - h_2 = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right)$$

Underground Flow



Flow generally driven by lake, river, etc.

$h_2 > h_1$ makes ground water flow at well possible

Darcy’s Law

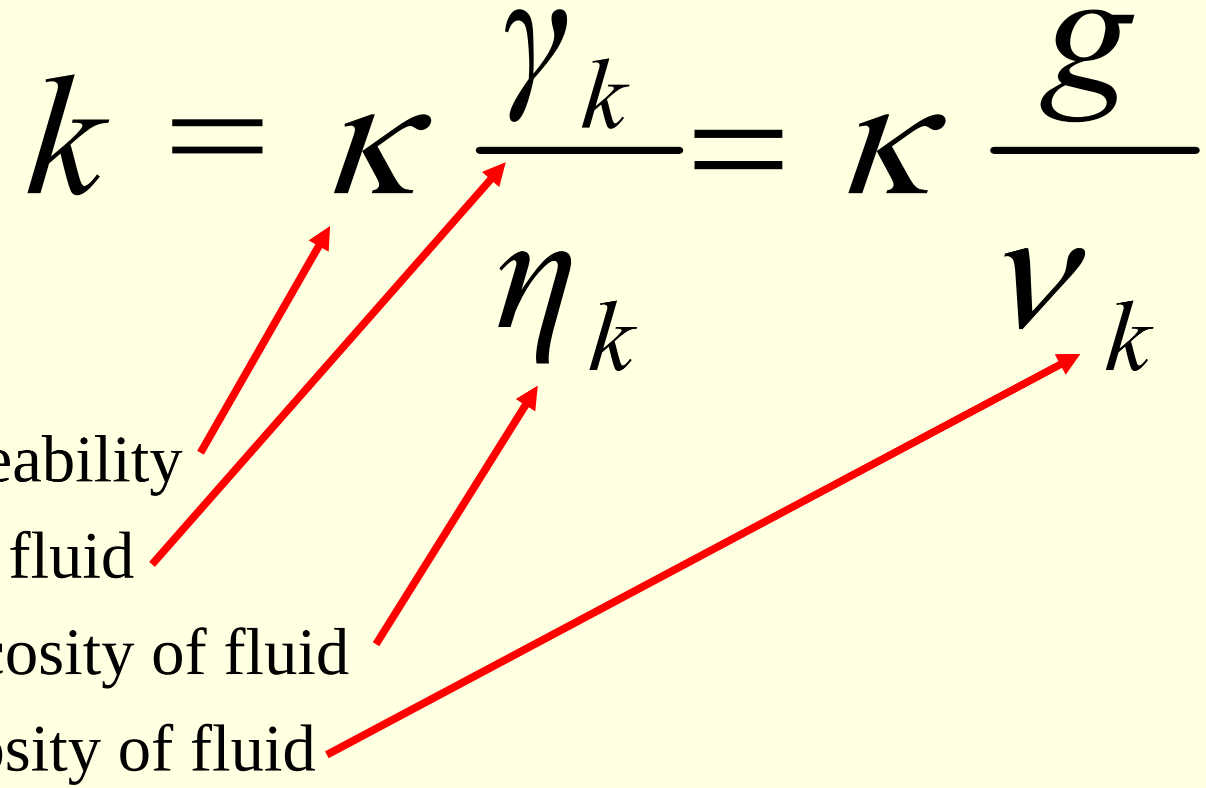
- Variables
  - k=coefficient of permeability
  - i=hydraulic gradient
- Assumptions
  - Laminar Flow
    - Generally applicable in soils due to low fluid flow velocities
    - Laminar flow generally exists when i < 5 and Re < 1
  - Soil is saturated

$$v = ki$$

$$q = kiA$$

Coefficient of Permeability or Hydraulic Conductivity

- Definition



$$k = \kappa \frac{\gamma_k}{\eta_k} = \kappa \frac{g}{\nu_k}$$

  - Physical permeability
  - Unit weight of fluid
  - Kinematic viscosity of fluid
  - Dynamic viscosity of fluid
- Absolute Permeability is significant with soils when different fluids are considered

Detailed Aspects of Permeability

- Kozeny-Carman Relationship

$$\kappa = cd^2 \frac{n^3}{(1-n)^2}$$

  - Difficult to determine permeability reliably from this relationship
  - Shows that a major variable in permeability is the diameter of the particles

- Seepage velocity

$$\bar{v} = \frac{v}{n}$$

  - Relationship between the specific discharge and the actual seepage velocity in the soil pores
  - Shows the impact of porosity (as does the Kozeny-Carman relationship) on permeability

Typical Values of k

Type of soil	k (m/s)
gravel	$10^{-3} - 10^{-1}$
sand	$10^{-6} - 10^{-3}$
silt	$10^{-8} - 10^{-6}$
clay	$10^{-10} - 10^{-8}$

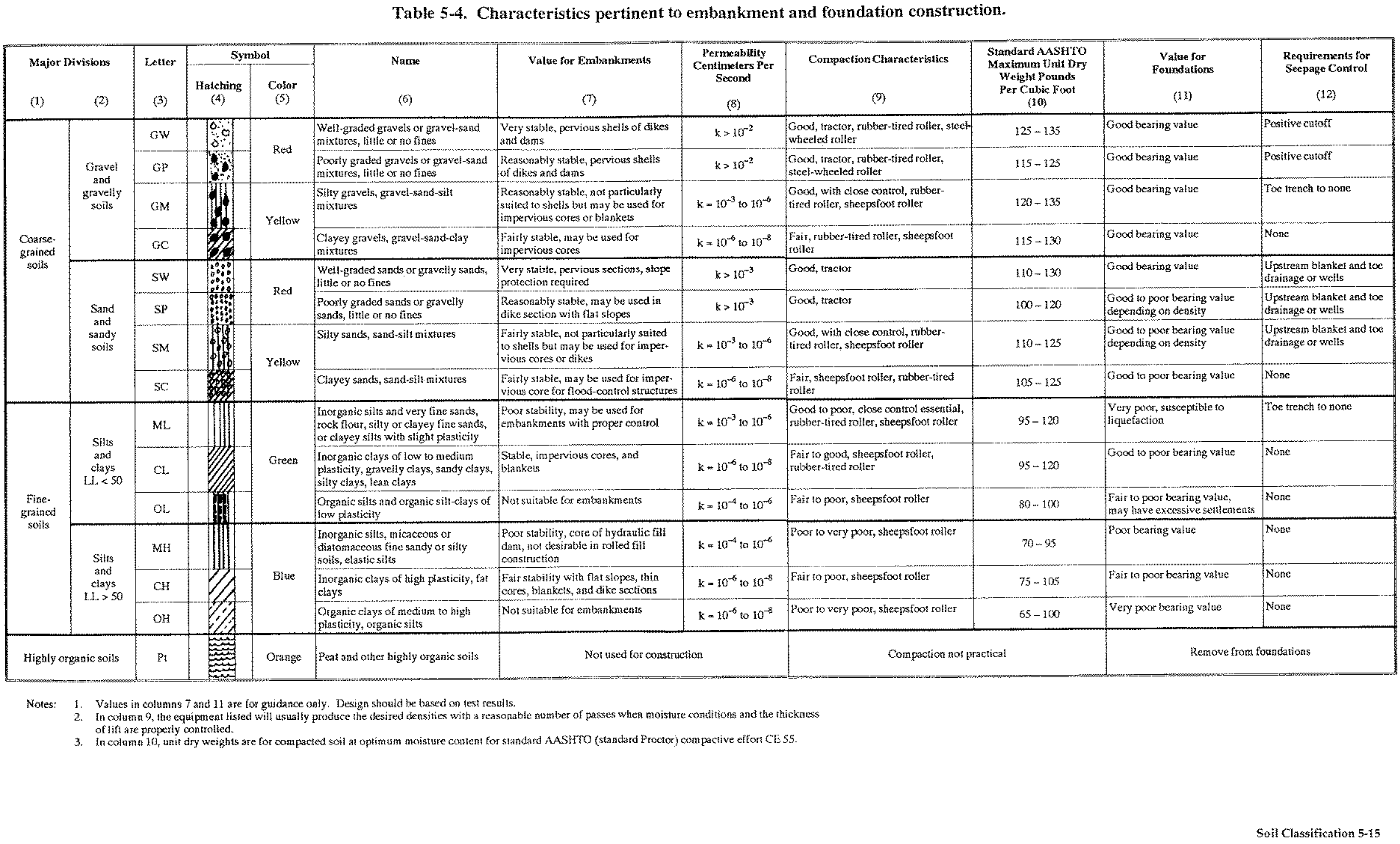


Table 5-4. Characteristics pertinent to embankment and foundation construction.

Major Divisions		Letter	Symbol		Name	Value for Embankments	Permeability Centimeters Per Second	Compaction Characteristics	Standard AASHTO Maximum Unit Dry Weight Pounds Per Cubic Foot	Value for Foundations	Requirements for Seepage Control
			Hatching (4)	Color (5)							
(1)	(2)	(3)			(6)	(7)	(8)	(9)	(10)	(11)	(12)
Coarse-grained soils	Gravel and gravelly soils	GW		Red	Well-graded gravels or gravel-sand mixtures, little or no fines	Very stable, pervious shells of dikes and dams	$k > 10^{-2}$	Good, tractor, rubber-tired roller, steel-wheeled roller	125 – 135	Good bearing value	Positive cutoff
		GP			Poorly graded gravels or gravel-sand mixtures, little or no fines	Reasonably stable, pervious shells of dikes and dams	$k > 10^{-2}$	Good, tractor, rubber-tired roller, steel-wheeled roller	115 – 125	Good bearing value	Positive cutoff
		GM		Yellow	Silty gravels, gravel-sand-silt mixtures	Reasonably stable, not particularly suited to shells but may be used for impervious cores or blankets	$k = 10^{-3}$ to 10^{-6}	Good, with close control, rubber-tired roller, sheepsfoot roller	120 – 135	Good bearing value	Toe trench to none
		GC			Clayey gravels, gravel-sand-clay mixtures	Fairly stable, may be used for impervious cores	$k = 10^{-6}$ to 10^{-8}	Fair, rubber-tired roller, sheepsfoot roller	115 – 130	Good bearing value	None
	Sand and sandy soils	SW		Red	Well-graded sands or gravelly sands, little or no fines	Very stable, pervious sections, slope protection required	$k > 10^{-3}$	Good, tractor	110 – 130	Good bearing value	Upstream blanket and toe drainage or wells
		SP			Poorly graded sands or gravelly sands, little or no fines	Reasonably stable, may be used in dike section with flat slopes	$k > 10^{-3}$	Good, tractor	100 – 120	Good to poor bearing value depending on density	Upstream blanket and toe drainage or wells
		SM		Yellow	Silty sands, sand-silt mixtures	Fairly stable, not particularly suited to shells but may be used for impervious cores or dikes	$k = 10^{-3}$ to 10^{-6}	Good, with close control, rubber-tired roller, sheepsfoot roller	110 – 125	Good to poor bearing value depending on density	Upstream blanket and toe drainage or wells
		SC			Clayey sands, sand-silt mixtures	Fairly stable, may be used for impervious core for flood-control structures	$k = 10^{-6}$ to 10^{-8}	Fair, sheepsfoot roller, rubber-tired roller	105 – 125	Good to poor bearing value	None
Fine-grained soils	Silts and clays LL < 50	ML		Green	Inorganic silts and very fine sands, rock flour, silty or clayey fine sands, or clayey silts with slight plasticity	Poor stability, may be used for embankments with proper control	$k = 10^{-3}$ to 10^{-6}	Good to poor, close control essential, rubber-tired roller, sheepsfoot roller	95 – 120	Very poor, susceptible to liquefaction	Toe trench to none
		CL			Inorganic clays of low to medium plasticity, gravelly clays, sandy clays, silty clays, lean clays	Stable, impervious cores, and blankets	$k = 10^{-6}$ to 10^{-8}	Fair to good, sheepsfoot roller, rubber-tired roller	95 – 120	Good to poor bearing value	None
		OL			Organic silts and organic silt-clays of low plasticity	Not suitable for embankments	$k = 10^{-4}$ to 10^{-6}	Fair to poor, sheepsfoot roller	80 – 100	Fair to poor bearing value, may have excessive settlements	None
	Silts and clays LL > 50	MH		Blue	Inorganic silts, micaceous or diatomaceous fine sandy or silty soils, elastic silts	Poor stability, core of hydraulic fill dam, not desirable in rolled fill construction	$k = 10^{-4}$ to 10^{-6}	Poor to very poor, sheepsfoot roller	70 – 95	Poor bearing value	None
		CH			Inorganic clays of high plasticity, fat clays	Fair stability with flat slopes, thin cores, blankets, and dike sections	$k = 10^{-6}$ to 10^{-8}	Fair to poor, sheepsfoot roller	75 – 105	Fair to poor bearing value	None
		OH			Organic clays of medium to high plasticity, organic silts	Not suitable for embankments	$k = 10^{-6}$ to 10^{-8}	Poor to very poor, sheepsfoot roller	65 – 100	Very poor bearing value	None
Highly organic soils		Pt		Orange	Peat and other highly organic soils	Not used for construction		Compaction not practical		Remove from foundations	

Notes:
1. Values in columns 7 and 11 are for guidance only. Design should be based on test results.
2. In column 9, the equipment listed will usually produce the desired densities with a reasonable number of passes when moisture conditions and the thickness of lift are properly controlled.
3. In column 10, unit dry weights are for compacted soil at optimum moisture content for standard AASHTO (standard Proctor) compactive effort CE 55.

Soil Classification 5-15

One Dimensional Example Illustration

- Given
  - River and Canal, parallel to each other
    - $\Delta h = 5\text{m}$
    - $L = 200$ m
    - $t_{stratum} = 2$ m
    - $k = 2$ m/day
- Find
  - Seepage loss per km of river-canal length

- Solution
  - $q = kiA$
  - $i = \Delta h/\Delta L = 5/200$
  - $q = (2)(5/200)(2) \times 1000)$
  - $q = 100\ m^3$/day/km length

Methods of Determining Permeability Coefficient

- Laboratory Methods
  - Constant Head Test (Cohesionless Soils)
  - Falling Head Test (Cohesive Soils)
- Empirical Correlations and “Typical” Values
- In-Situ Methods
- Pumping Tests
- Consolidation Tests
- Permeability is the most difficult soil property to determine accurately, especially in the laboratory

- Laboratory Tests
  - Constant Head Test
    - Direct measure of permeability using Darcy's Law
    - Suitable for cohesionless soils with permeabilities > 10 x 10^{-4} cm/sec
  - Falling Head Test
    - Indirect measurement of permeability using time of flow
    - Suitable for cohesive soils with permeabilities < 10 x 10^{-4} cm/sec

Constant Head Test

- The simplest of all methods for determining the coefficient of permeability
- This test is performed by measuring the quantity of water, Q, flowing through the soil specimen, the length of the soil specimen, L, the head of water, h, and the elapsed time, t. The head of water is kept constant throughout the test.

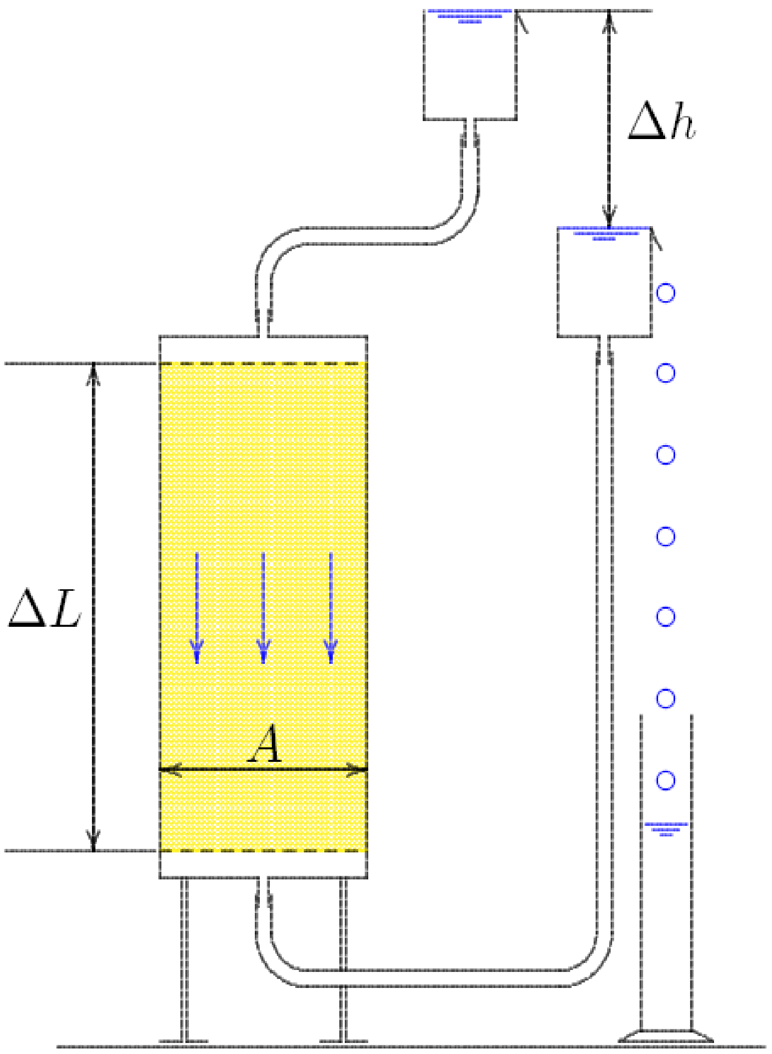


Figure 7.1: Permeability test.

Computation

- Compute the coefficient of permeability, k for laboratory test:
  - k_{20} = coefficient of permeability, cm/sec at 20° C
  - Q= quantity of flow, cm^3
  - L = length of specimen over which head loss is measured, cm.
  - R_T = temperature correction factor for viscosity of water
  - h = loss of head in length, L, or difference in piezometer readings = $h_1 - h_2$, cm
  - A = cross-sectional area of specimen, cm^2
  - t = elapsed time, sec.

$$v = ki$$

$$\frac{Q}{At} = \frac{k_{20}}{R_T}\frac{\Delta h}{\Delta L}$$

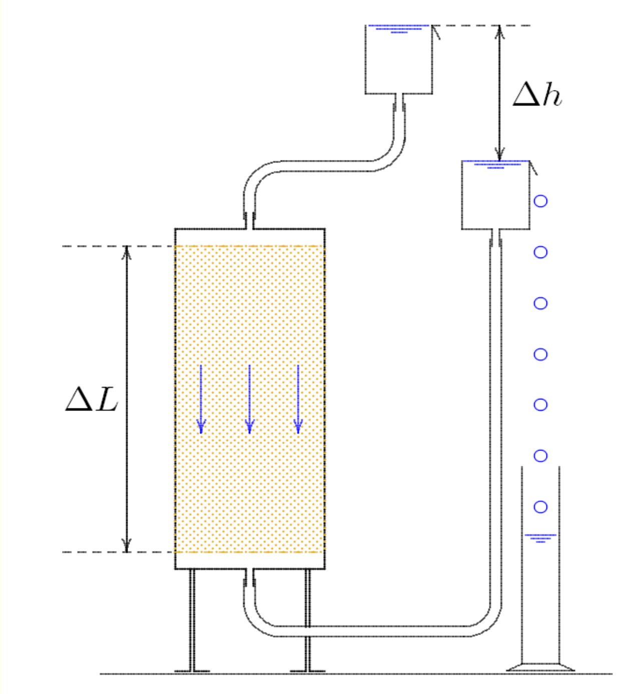
Solving,

$$k_{20} = \frac{Q \Delta L R_T}{\Delta h A t}$$

Sample Case: Constant Head Permeameter

- Given:
  - Sand Sample
    - 15 cm high
    - 5.5 cm diameter (round)
  - Test Results
    - Measured head across sample: 40 cm
    - Time of flow: 6 sec.
    - Amount of water discharged: 400 cc
    - Assume test run at 20° C (no temperature/viscosity correction)
  - Find: Coefficient of Permeability

- Governing Equation
  - q = kiA
- Definition of Variables in Governing Equation
  - Flow rate q = Q/t
  - Sample area A = $\pi D^2/4$
  - Hydraulic Gradient i = $\Delta h/\Delta L$



Sample Case: Constant Head Permeameter

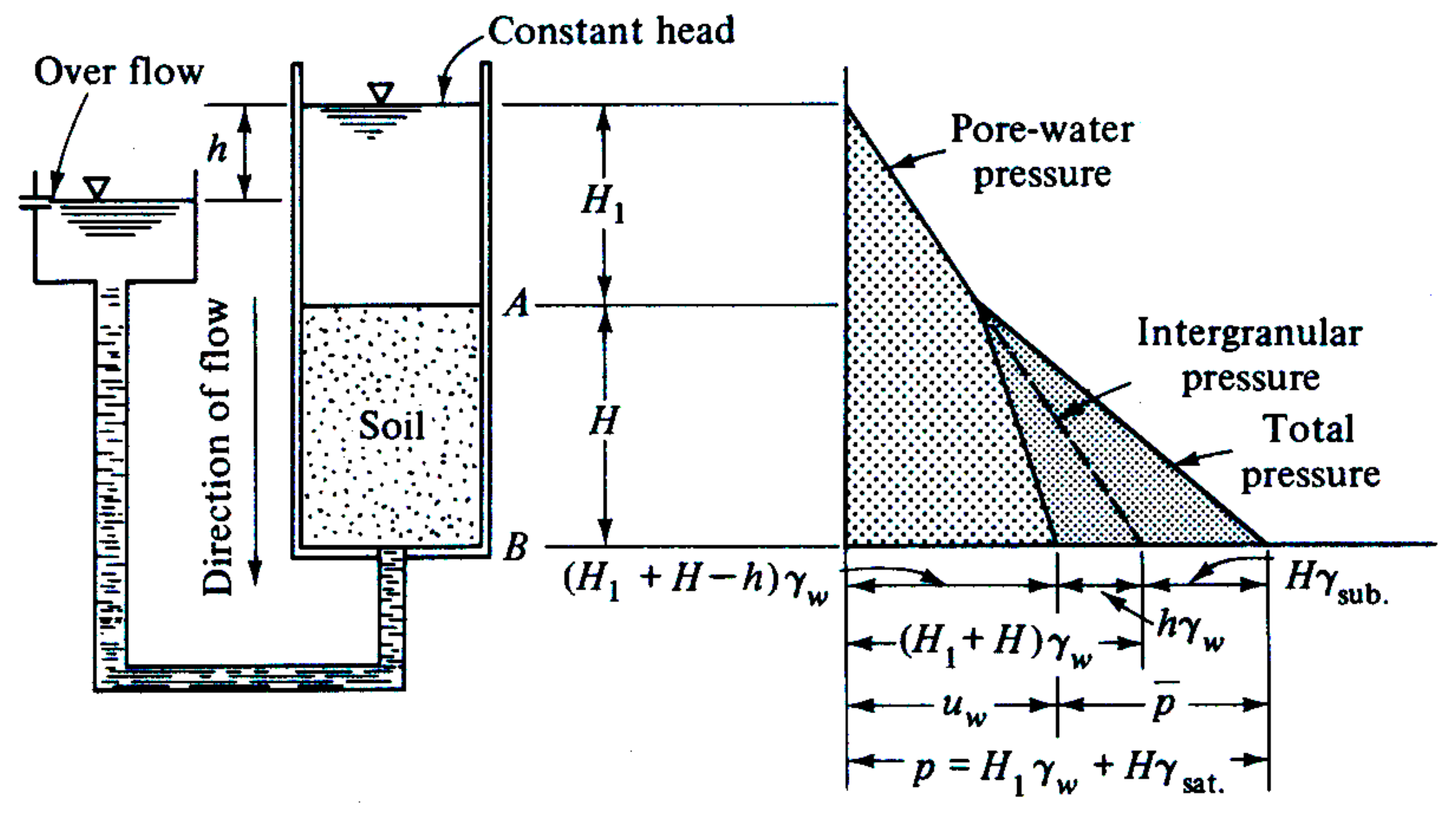
$$k_{20} = \frac{QLR_T}{hAt}$$

$$A = \frac{\pi D^2}{4}; R_T = 1$$

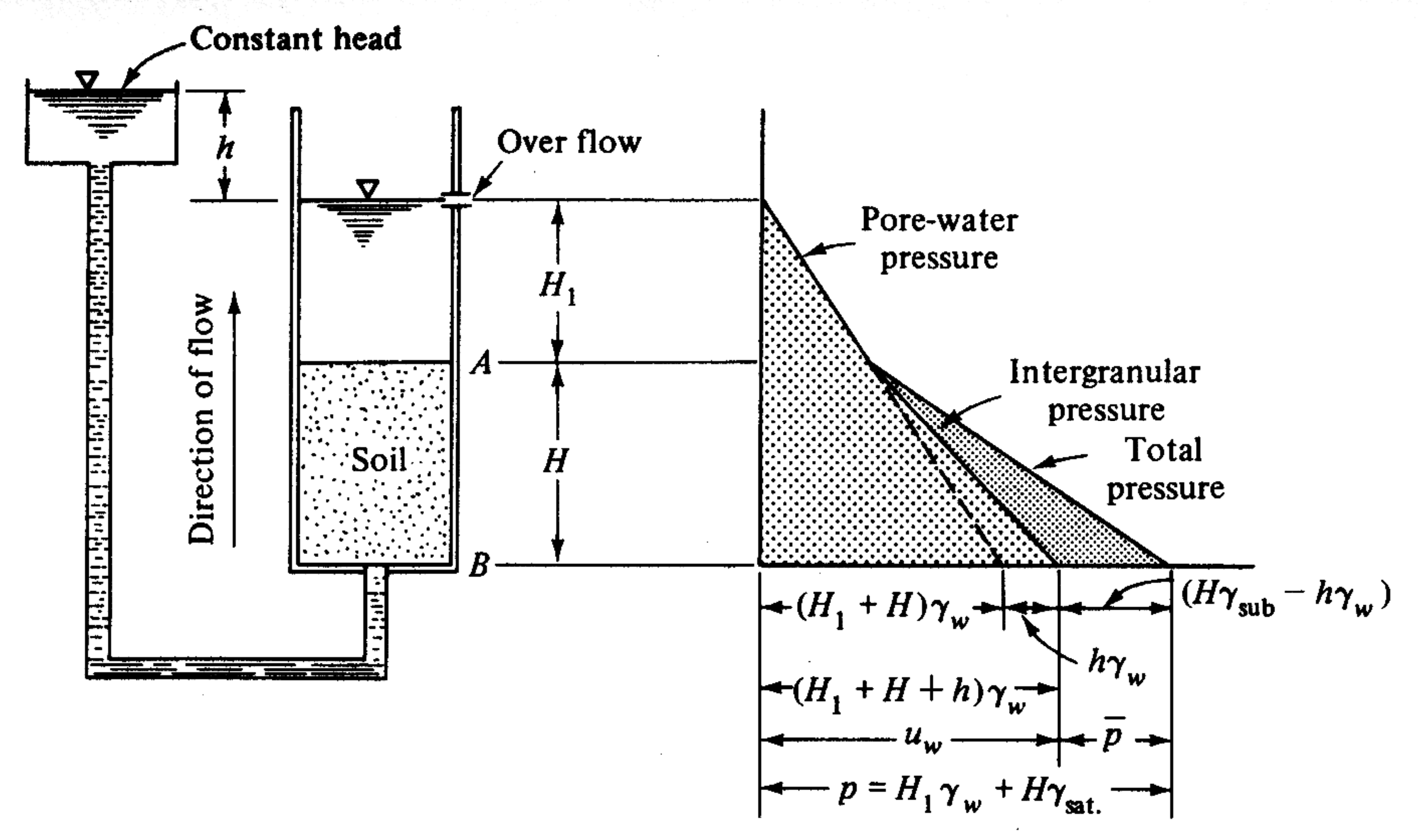
$$k_{20} = \frac{4QL}{\pi D^2 ht}$$

- Rearrange Equation
- Substitute Variables and solve:
- $k = ((4)(400)(15))/((3.1416)(5.5)^2(40)(6)) = 1.052$ cm/sec

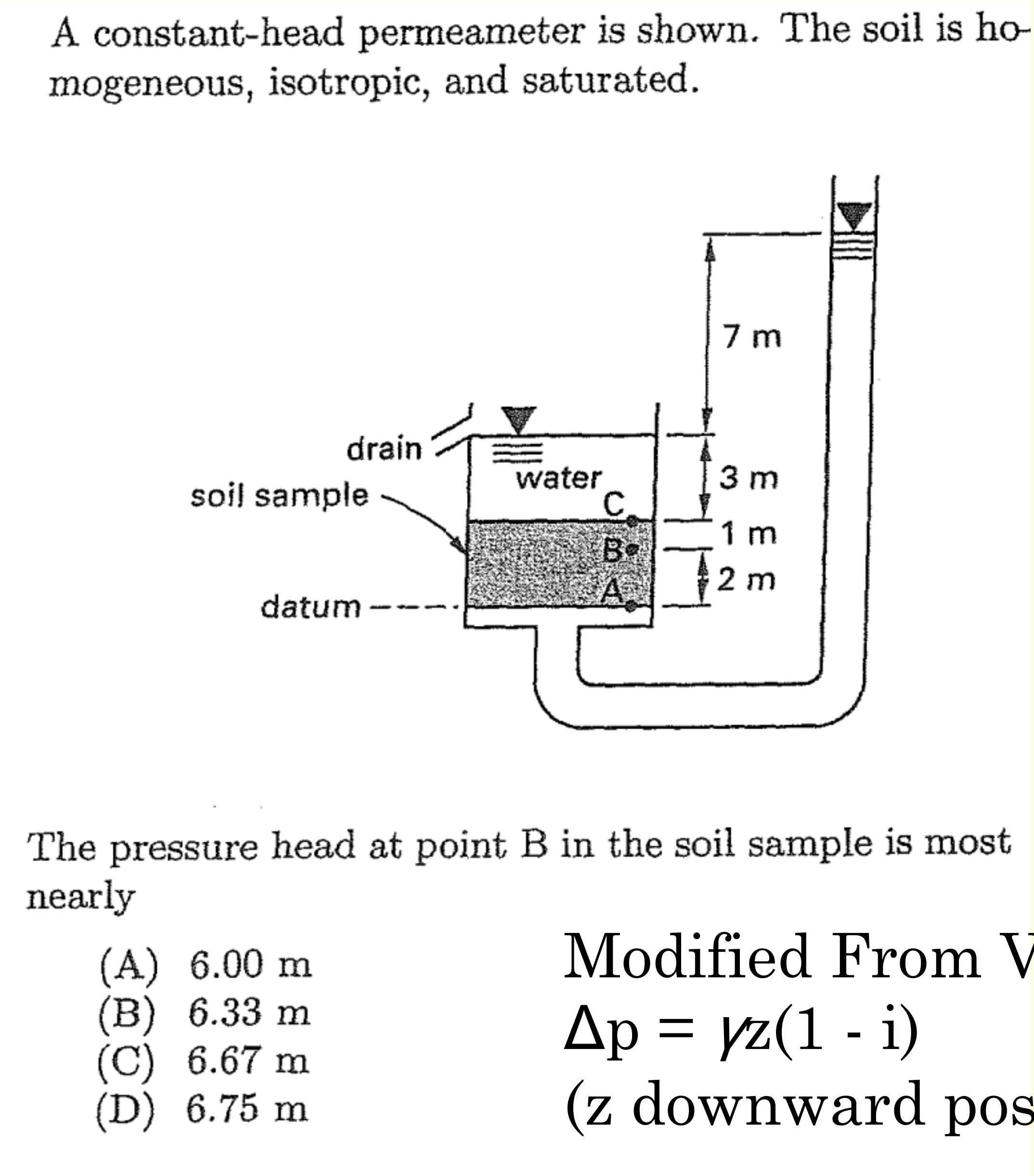
Soil and Water Pressure with Downward (Vertical) Flow



Soil and Water Pressure with Upward (Vertical) Flow



Example of Upward Flow



A constant-head permeameter is shown. The soil is homogeneous, isotropic, and saturated.

The pressure head at point B in the soil sample is most nearly

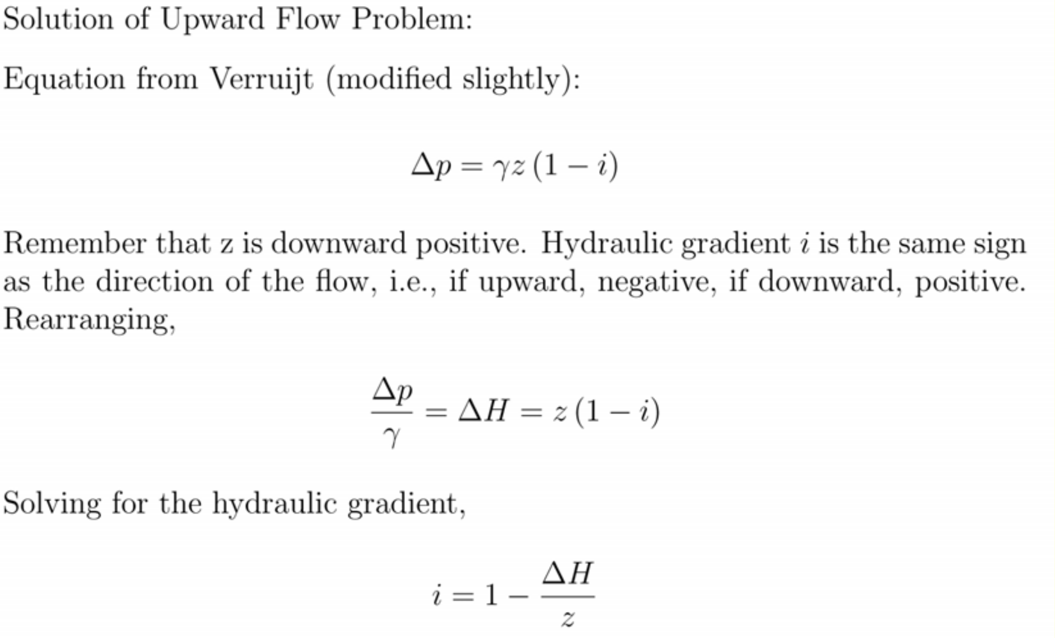
- (A) 6.00 m
- (B) 6.33 m
- (C) 6.67 m
- (D) 6.75 m

Modified From Verruijt:

$\Delta p = \gamma z(1 - i)$

(z downward positive)

Example of Upward Flow



Solution of Upward Flow Problem:

Equation from Verruijt (modified slightly):

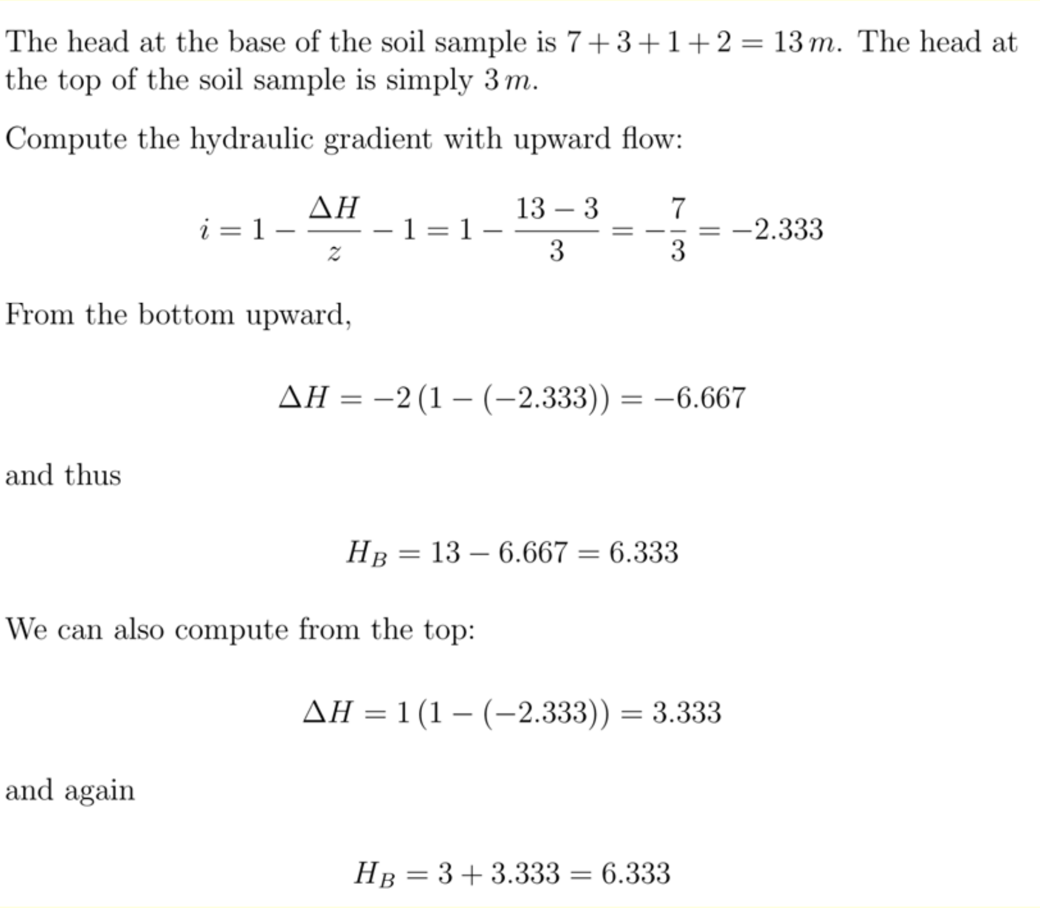
$$\Delta p = \gamma z \left(1 - i\right)$$

Remember that z is downward positive. Hydraulic gradient i is the same sign as the direction of the flow, i.e., if upward, negative, if downward, positive. Rearranging,

$$\frac{\Delta p}{\gamma} = \Delta H = z \left(1 - i\right)$$

Solving for the hydraulic gradient,

$$i = 1 - \frac{\Delta H}{z}$$



The head at the base of the soil sample is $7 + 3 + 1 + 2 = 13\,m$. The head at the top of the soil sample is simply $3\,m$.

Compute the hydraulic gradient with upward flow:

$$i = 1 - \frac{\Delta H}{z} - 1 = 1 - \frac{13 - 3}{3} = -\frac{7}{3} = -2.333$$

From the bottom upward,

$$\Delta H = -2 \left(1 - (-2.333)\right) = -6.667$$

and thus

$$H_B = 13 - 6.667 = 6.333$$

We can also compute from the top:

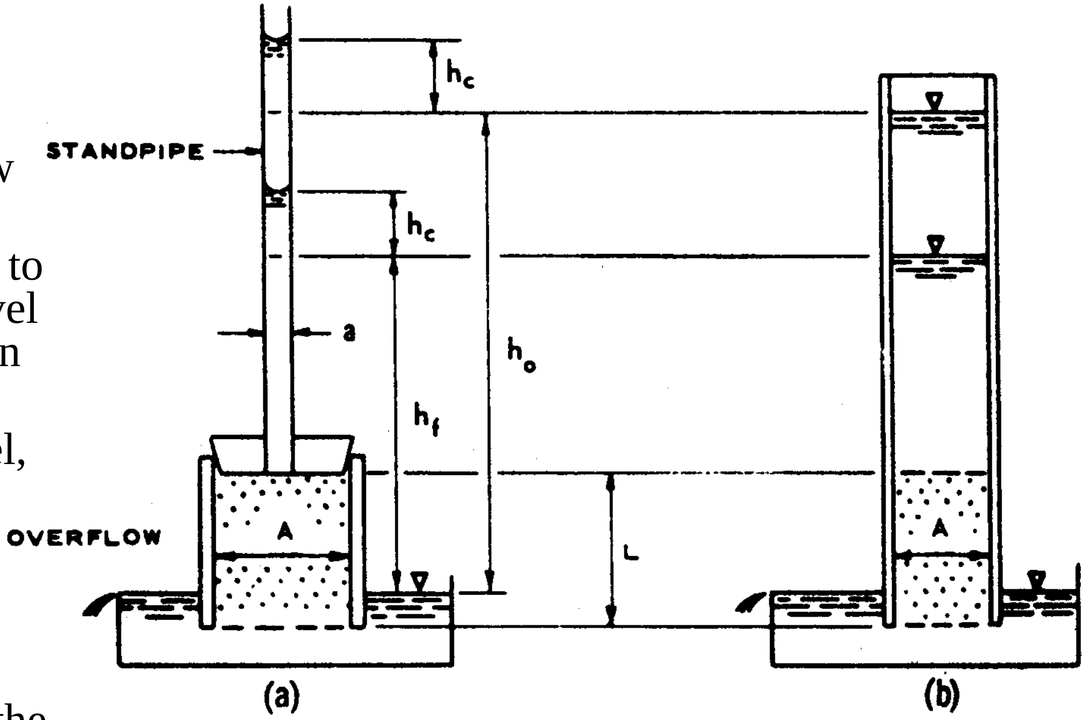
$$\Delta H = 1 \left(1 - (-2.333)\right) = 3.333$$

and again

$$H_B = 3 + 3.333 = 6.333$$

Falling Head Test

- A clay sample is enclosed by a circular ring, placed in a container filled with water.
- The lower end of the sample is in open connection with the water in the container, through a porous stone below the sample.
- At the top of the sample it is connected to a thin glass tube, in which the water level is higher than the constant water level in the container.
- Because of this difference in water level, water will flow through the sample, in very small quantities, but sufficient to observe by the lowering of the water level in the thin tube.
- In this case the head difference h is not constant, because no water is added to the system, and the level ho is gradually reduced to hf.



Computations

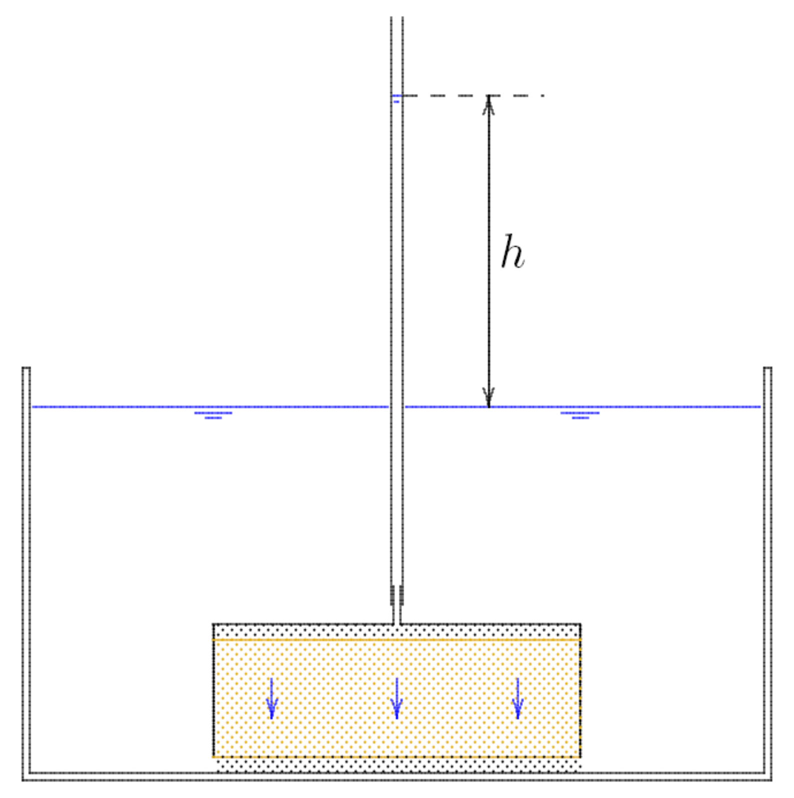
- Compute the coefficient of permeability, k

$$k = \frac{aL}{At}\left(\ln\frac{h_o}{h_f}\right)R_T$$

  - a = inside area of standpipe, cm²
  - A = cross-sectional area of specimen, cm²
  - L = length of specimen, cm
  - t = elapsed time (t_f - t_o), sec
  - h_o = height of water in standpipe above discharge level at time t_o, cm
  - h_f = height of water in standpipe above discharge level at time t_f, cm
  - R_T = temperature correction factor for viscosity of water obtained from , degrees C.

Falling Head Permeameter Sample Problem

- Given: Clay Sample
  - Sample Diameter: 2.5”
  - Sample Thickness: 1”
  - Standpipe Diameter: 1.7 mm
  - Initial elevation of water = 32 cm
  - Time of Test = 395 seconds
  - Final elevation of water = 30 cm
- Find
  - Coefficient of Permeability of clay



Falling Head Permeameter Sample Problem

- Governing Equation

$$k = \frac{L}{t_f - t_o}\frac{a}{A}\ln\left(\frac{h_o}{h_f}\right)$$

- Definition of variables in governing equation

$$A = \frac{1}{4}\pi D_o^2$$

$$a = \frac{1}{4}\pi d_o^2$$

- Substitution of defined variables

$$k = \frac{L}{t_f - t_o}\frac{d_o^2}{D_o^2}\ln\left(\frac{h_o}{h_f}\right)$$

- Substitution of input quantities

$$k = \frac{2.54}{395 - 0}\frac{0.17^2}{(2.5 \times 2.54)^2}\ln\left(\frac{32}{30}\right)$$

$$k = 2.974 \times 10^{-6}\, cm / sec$$

Questions?