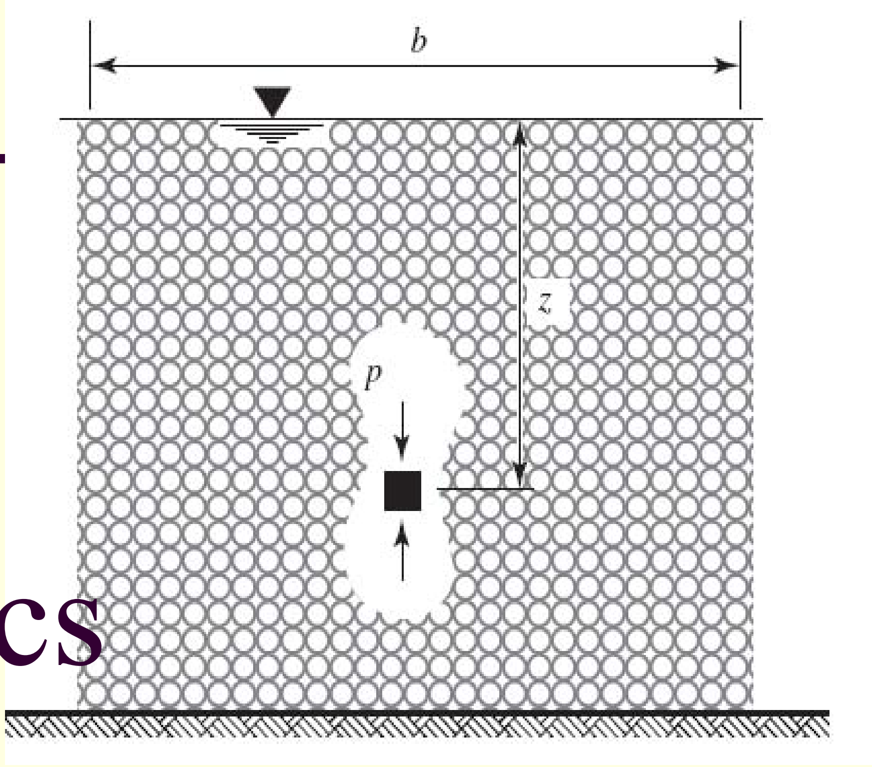
ENCE 3610
Soil Mechanics



Lecture 5
Effective (Intergranular) Stress in Soils
Capillary Rise in Soils

Stresses in Soils

- Geostatic and Hydrostatic Stresses
  - Total Stress
  - Pore Water Pressure
  - Effective Stress and Overburden Pressure (po)
  - Upward and Downward Seepage Stresses
  - Lateral Earth Pressures
- External Load Stresses
  - Elastic Methods (Boussinesq, etc.)
  - Empirical Methods (2:1)
- Combined Stresses and Mohr's Circle
  - Use of Mohr's Circle (principal stresses, etc.)
  - Theory of elasticity and plasticity
- General Comments
  - The principal source of soil stress is caused by the weight of the soil (adjusted usually by the pore water pressure) above the point in the soil under examination
  - Additional stresses can and are induced by load from the soil surface, such as footings, deep foundations, embankments and other weight-bearing structures

Hydrostatic and Total Stress

- Hydrostatic Stress
  - The stress induced by the weight of the water at a given depth

$$u = \gamma_w z$$

  - Assumes water is at rest and not experiencing forces other than static gravity
  - Also exists in the pores of the soil as it does in free water, thus the name "pore water pressure" (but pore water pressure doesn't have to be hydrostatic)

- Total Stress
  - The stress produced at any point by the overburden pressure of the soil plus any applied loads (equation does not include loads)

$$\sigma_{total} = \sum_{i=1}^{n} H_i \gamma_{t_i}$$

- Generally ignores the effect of pore water pressure
- Applies to any soil in any state of saturation

Pore Water Pressure

- Pressure of water within the pores or voids of the soil
- Can be caused by:
  - Hydrostatic pressure, either direct from above or from downflow of water
  - Capillary action
  - Seepage
  - Pressure from applied loads compressing trapped water
- Water has a buoyant effect on soils under the water table
- For hydrostatic conditions, this includes all soils under the water table
- The buoyant effect of the water equals the weight of the water the soil displaces
- This reduces the effective unit weight of the soil for the purposes of calculating effective stress

Effective (Intergranular) Stress

$$\sigma = \frac{W_s + \left(zb^2\gamma_w - V_s\gamma_w\right)}{b^2}$$

$$u = z\gamma_w$$

$$\sigma = \frac{W_s + \left(ub^2 - V_s\gamma_w\right)}{b^2} = \frac{W_s - V_s\gamma_w}{b^2} + u$$

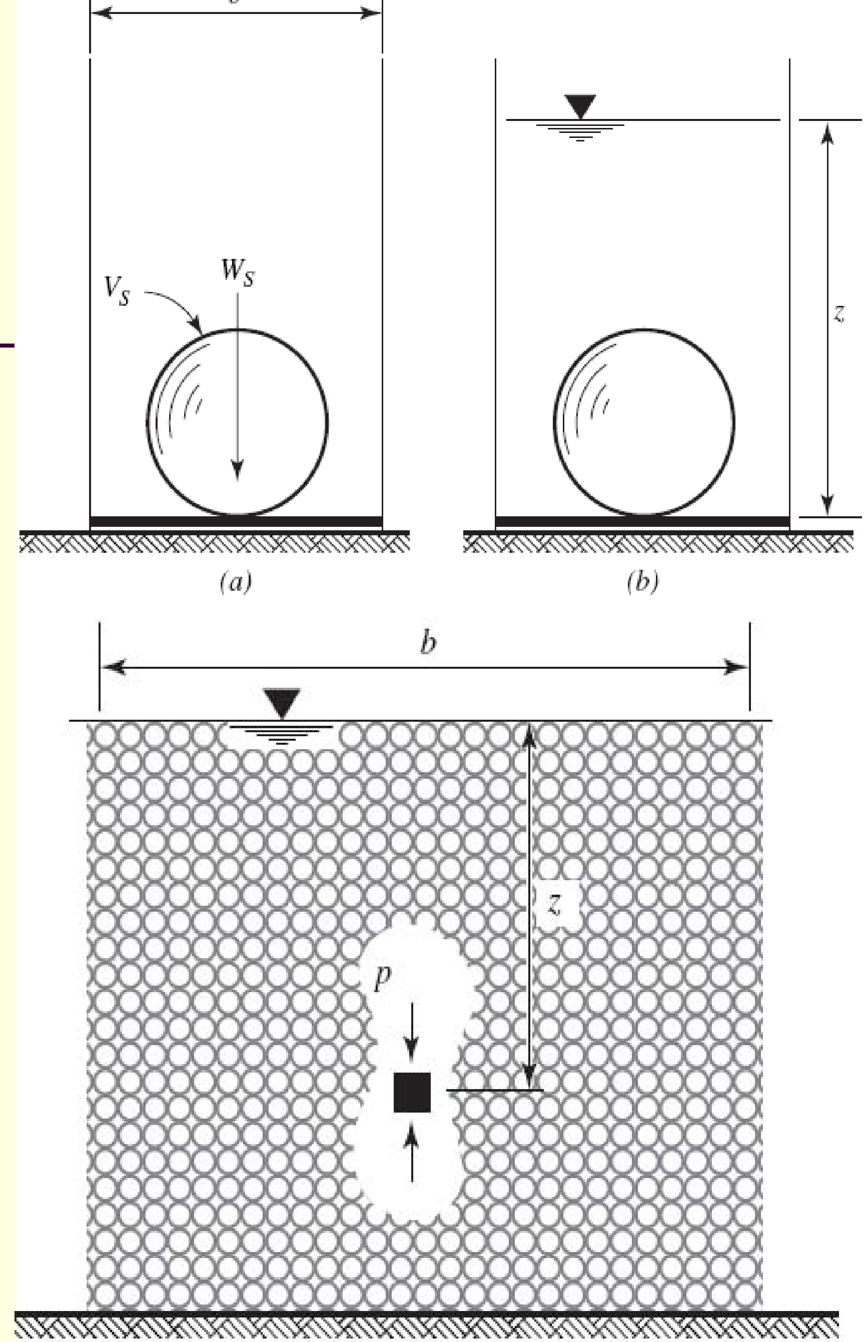
$$\sigma' = \sigma - u \text{ (Definition of Effective Stress)}$$

$$W_s = V_s G_s \gamma_w$$

$$\sigma'_z = \frac{V_s G_s \gamma_w - V_s\gamma_w}{b^2} = \frac{V_s\gamma_w}{b^2}\left(G_s - 1\right)$$

$$V_s = \frac{V}{1+e}\,;\, z = \frac{V}{b^2}$$

$$\sigma'_z = z\gamma_w \frac{G_s - 1}{e+1} = z\gamma_w\left[\frac{G_s + e}{e+1} - \frac{e+1}{e+1}\right] = z\left[\gamma_{sat} - \gamma_w\right] = z\gamma_{sub}$$



Saturated, hydrostatic conditions

Homogeneous soil layer

Effective Stress—General Expression

- The total stress minus the pore water pressure

$$\sigma_z' = p_o = \sum_{i=1}^{n} H_i \gamma_{t_i} - u$$

- Equation only considers the load of the soil itself
- One of the most important concepts in soil mechanics

- As depth increases, soil stress generally increases because of increasing H
- Varying unit weight will vary increase in effective stress
- Above water table, pore water pressure is generally not considered (except for capillary condition)

Methods for Computation of Effective Stress

- "Pore Water plus Total Stress" Method
  - Tabulate heights of layers and unit weights (saturated and unsaturated of each)
  - Note location of water table
  - Use this equation to compute effective stress:

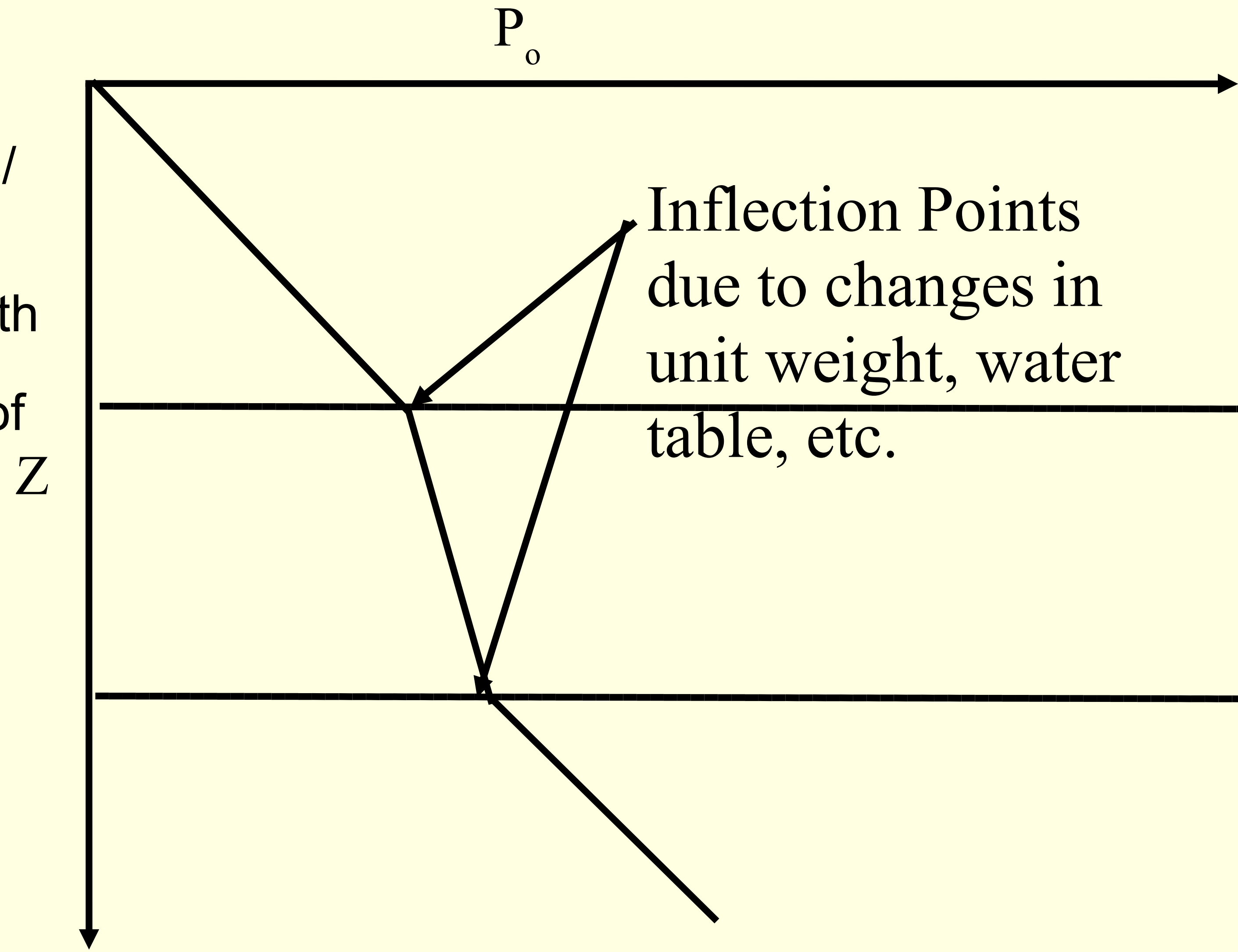
$$\sigma_z' = \sum_{i=1}^{n} H_i \gamma_{t_i} - u$$

- Submerged Weight Method
  - Tabulate layer heights and unit weights as with other method
  - Layers above water table use unsaturated (wet) unit weight
  - Layers below water table use submerged unit weight
  - Add as with other method

$$\gamma_{sub} = \gamma_{sat} - \gamma_w \approx \frac{\gamma_{sat}}{2}$$

P_o Diagram

- Useful tool to visualise the increase of effective stress/ overburden pressure as a function of depth and to clearly see the effect of the pore water pressure
- Can also be used in some cases to illustrate the changes that take place with applied loads and other conditions

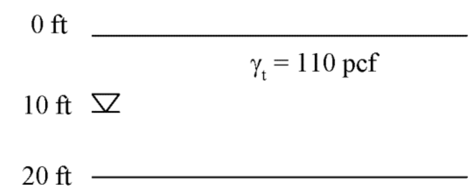


Methods of Plotting P_o Diagrams

- Manual Plotting
  - Find points of inflection (changes in unit weight, water table level, other changes in pore water pressure, etc.)
  - Compute effective stress/overburden pressure at each point using standard equations
  - Join points with lines
  - In some cases, pore water pressure functions are not linear, so lines cannot accurately be used in P_o diagrams
- Spreadsheet Plotting
  - Divide up soil into layers of constant effective stress function with depth
  - Compute increase in effective stress for each layer
  - Successively add the increments of increasing (usually) effective stress and create a set of data points at the bottom of each layer
  - Use the data set to plot the P_o diagram

P_o Example 1

Example 2-1: Find p_o at 20 ft below ground in a sand deposit with a total unit weight of 110 pcf and the water table 10 ft below ground. Assume $\gamma_t = \gamma_{sat}$. Plot p_t and p_o versus depth from 0 ft – 20 ft.



Solution:

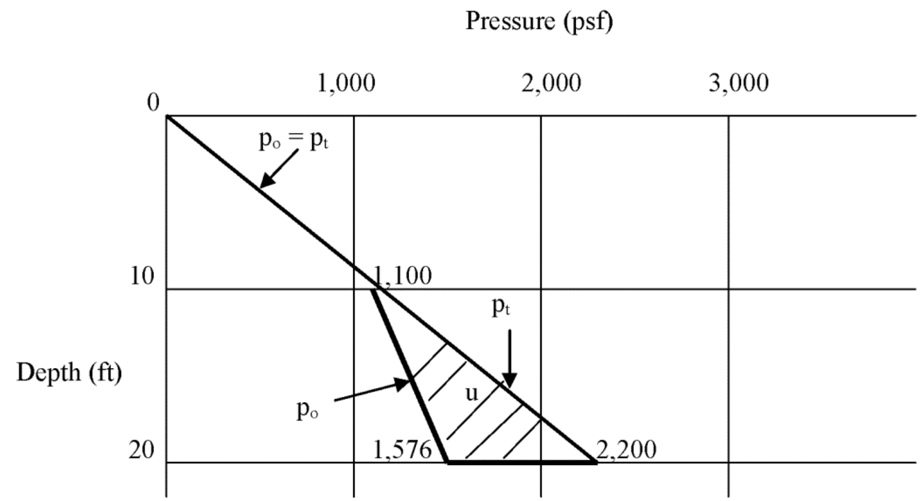
From Equation 2-13, $p_o = p_t - u$

p_t @ 10 ft = p_o @ 10 ft = 10 ft × 110 pcf = 1,100 psf

p_t @ 20 ft = p_t @ 10 ft + (10 ft × 110 pcf) = 2,200 psf

u @ 20 ft = 10 ft × 62.4 pcf = 624 psf

p_o @ 20 ft = p_t @ 20 ft - u @ 20 ft = 2,200 psf – 624 psf = 1,576 psf



Pressure Diagram

A plot of effective overburden pressure versus depth is called a "p_o – diagram" and is used throughout all aspects of geotechnical testing and analysis.

Figure 2-7. Example calculation of a p_o-diagram.

Effective Stress (p_0) Example 2

- Given
  - Idealized soil profile as follows:

Table A.4-1
Unit weights of soils in idealized profile (Boring UDH BAF-4)
(Assume unit weight of water = 60 pcf)

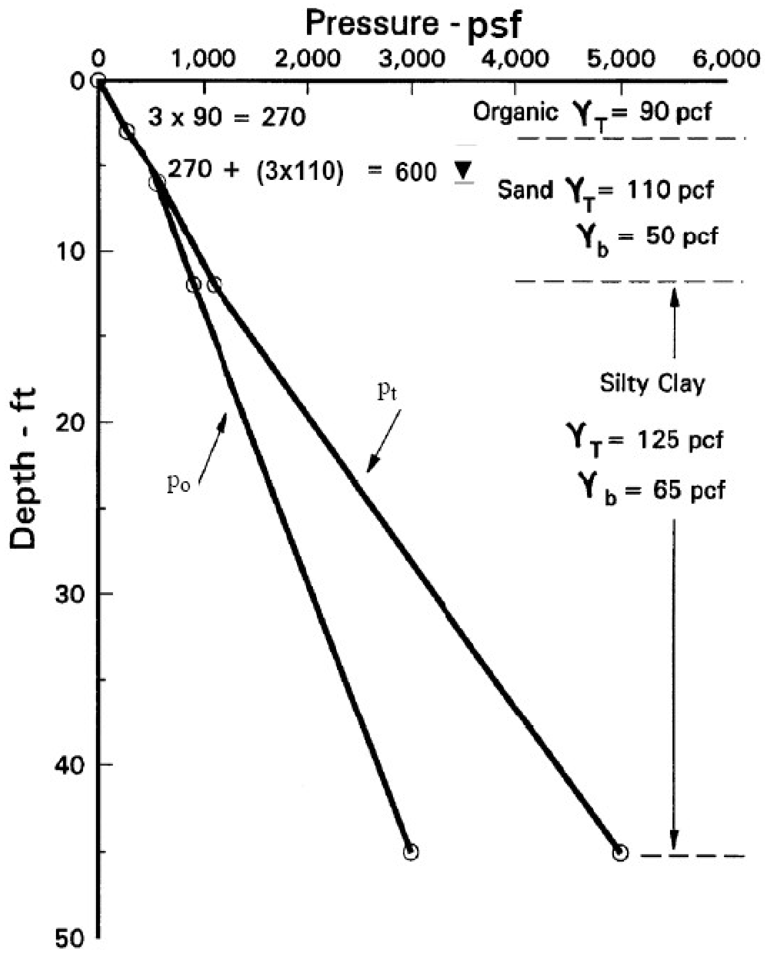
Soil stratum	Inclusive Depth (ft.)	Total unit weight (γ_t) pcf	Saturated unit weight (γ_{sat}) pcf	Buoyant unit weight (γ_b) pcf
Organics	0 - 3	90	-	-
Sand	3 - 10	110	110	50
Silty clay	10 - 45	125	125	65

  - Water table is at a depth of 6' below the ground surface
  - Note submerged (buoyant) unit weights

- Find
  - p_0 diagram for soil profile
  - This should include a plot of both total and effective stresses
  - Assume hydrostatic conditions

Effective Stress (p_0) Example 2

- Solution
  - Diagram



- Solution
  - Tabular results and calculations

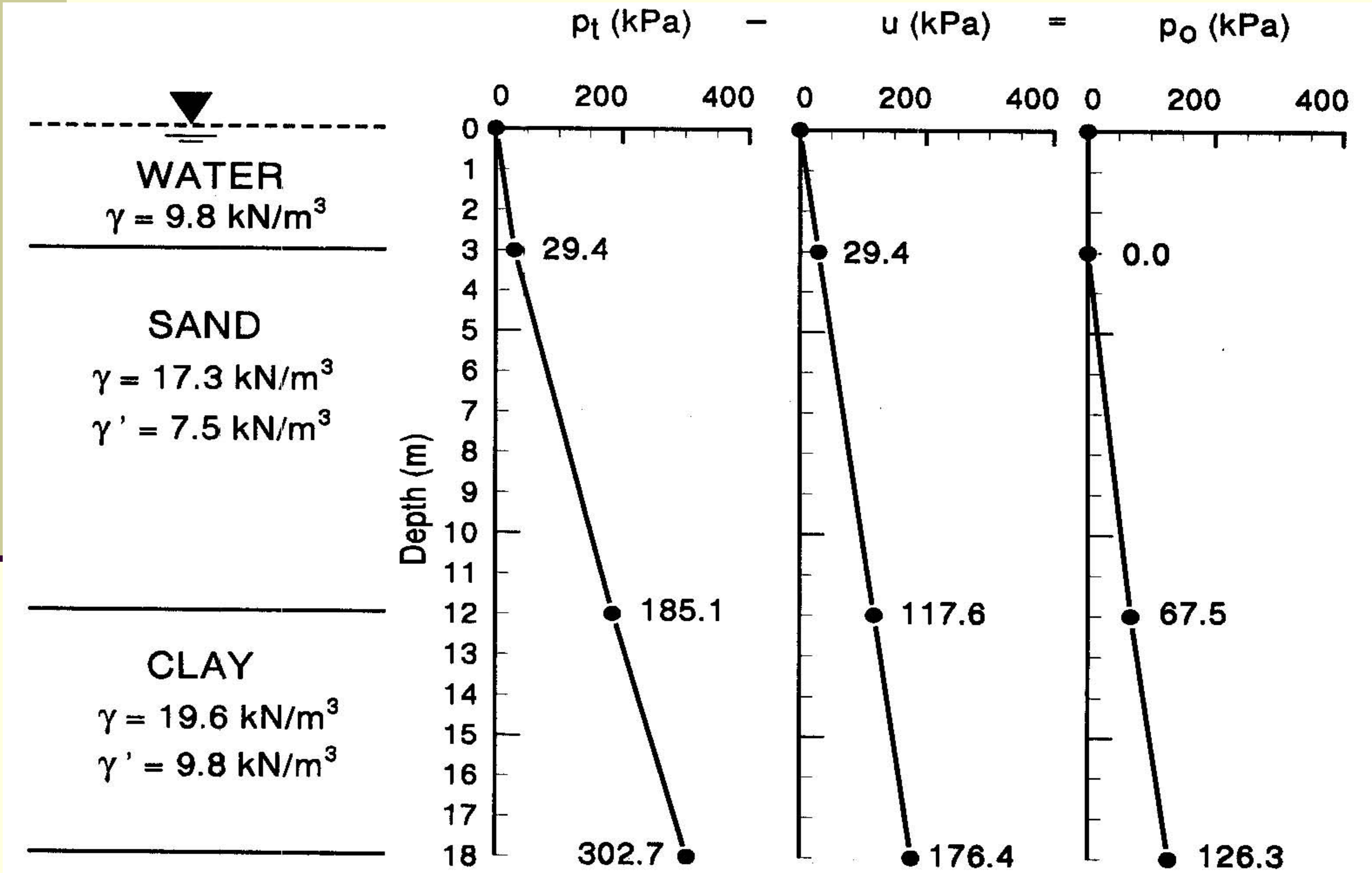
Table A.4-2

Computations for construction of p_o diagram (Boring UDH BAF-4)

(The ground water table [GWT] is located at a depth of 6-ft below the surface)

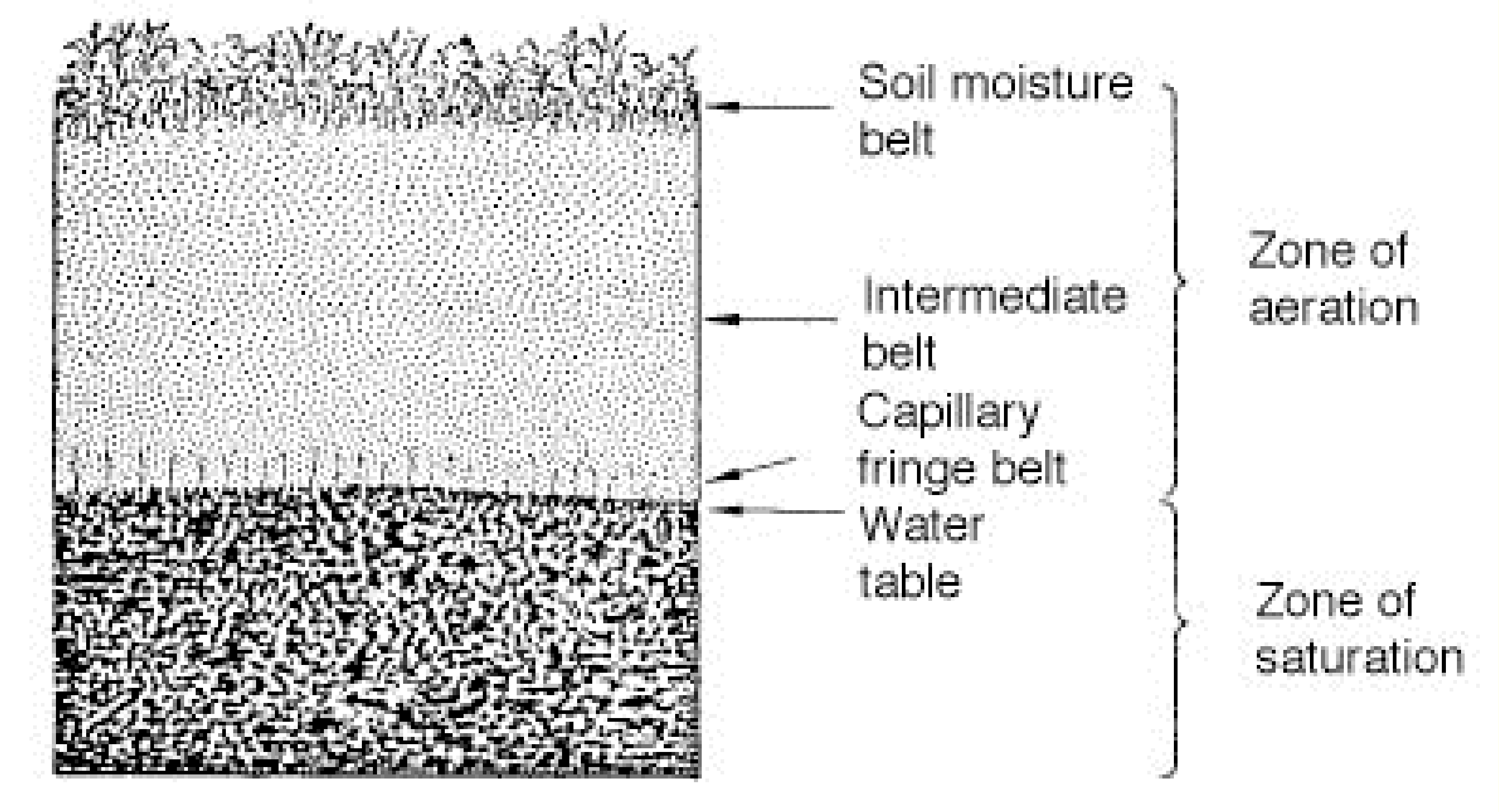
Depth (ft) to a boundary	Total (p_t) geostatic vertical pressure (psf)	Effective (p_o) geostatic vertical pressure (psf)	Hydrostatic (p_w) pore water pressure (psf)
3	3-ft x 90 pcf = 270 psf	3-ft x 90 pcf = 270 psf	$p_t - p_o = 0$ (above GWT)
6	270 psf + 3 ft x 110 pcf = 600 psf	270 psf + 3 ft x 110 pcf = 600 psf	$p_t - p_o = 0$ (at GWT)
10	600 psf + 4 ft x 110 pcf = 1,040 psf	600 psf + 4 ft x 50 pcf = 800 psf	1,040 psf – 800 psf = 240 psf or 4 ft x 60 pcf = 240 psf
45	1,040 psf + 35 ft x 125 pcf = 5,415 psf	800 psf + 35-ft x 65 pcf = 3,075 psf	5,415 psf – 3,075 psf = 2,340 psf or 39-ft x 60 pcf = 2,340 psf

P_o Example 3



pt (kPa)
–
u (kPa)
=
po (kPa)
WATER
γ = 9.8 kN/m3
SAND
γ = 17.3 kN/m3
γ ' = 7.5 kN/m3
CLAY
γ = 19.6 kN/m3
γ ' = 9.8 kN/m3
Depth (m)
0
200
400
29.4
185.1
302.7
117.6
176.4
0.0
67.5
126.3

Groundwater Zones in Soil



Capillarity and Capillary Rise in Soils

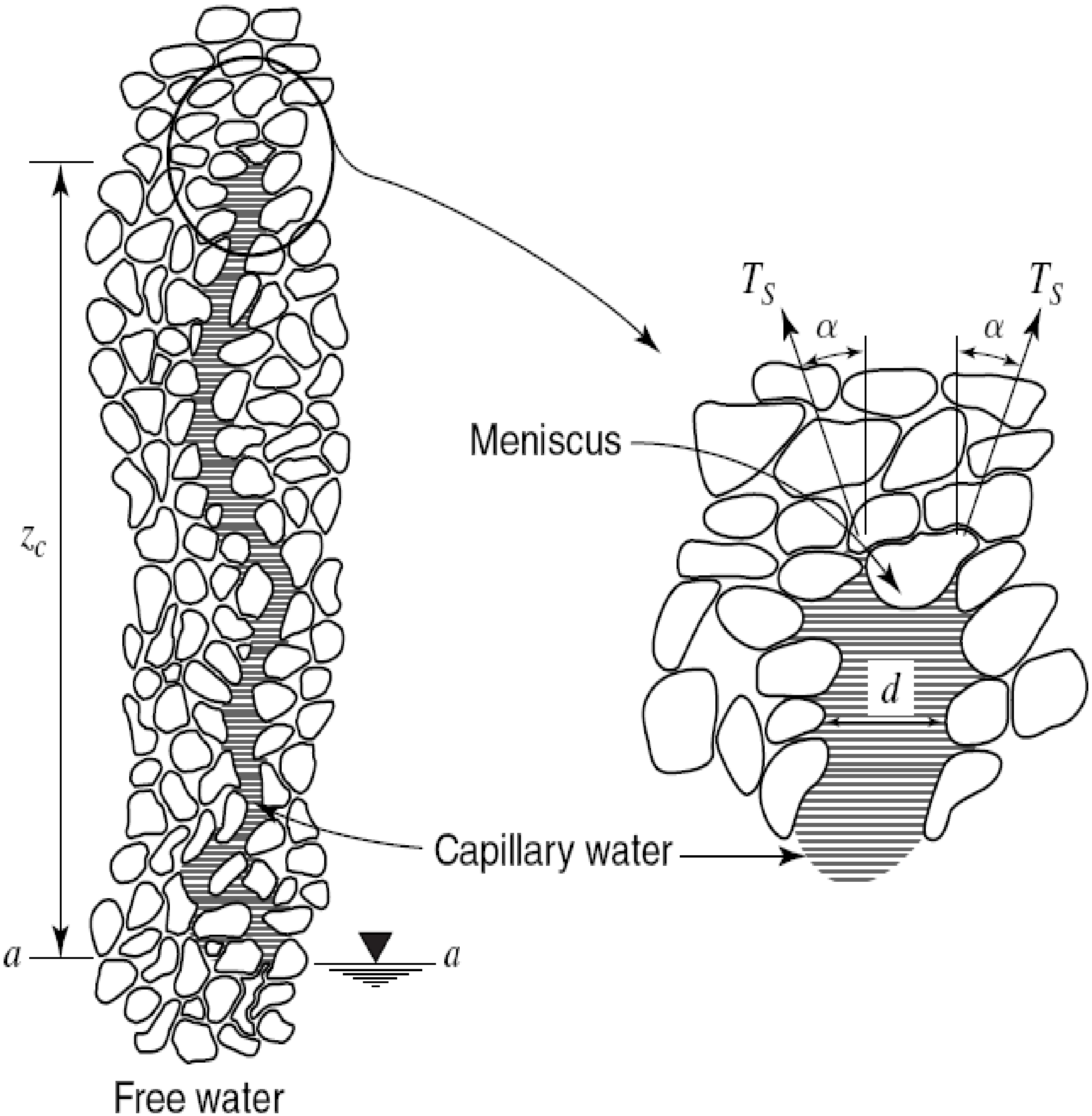


Figure 5.4 Capillary rise in soils.

- Result of surface tension in water between the water itself and a solid surface it touches (glass tube, soil particles, etc.)
- Capillary rise takes place when the surface tension in the water is greater than the force of gravity of the water under it
- Capillary rise stops when the force of the water equals the weight supporting it

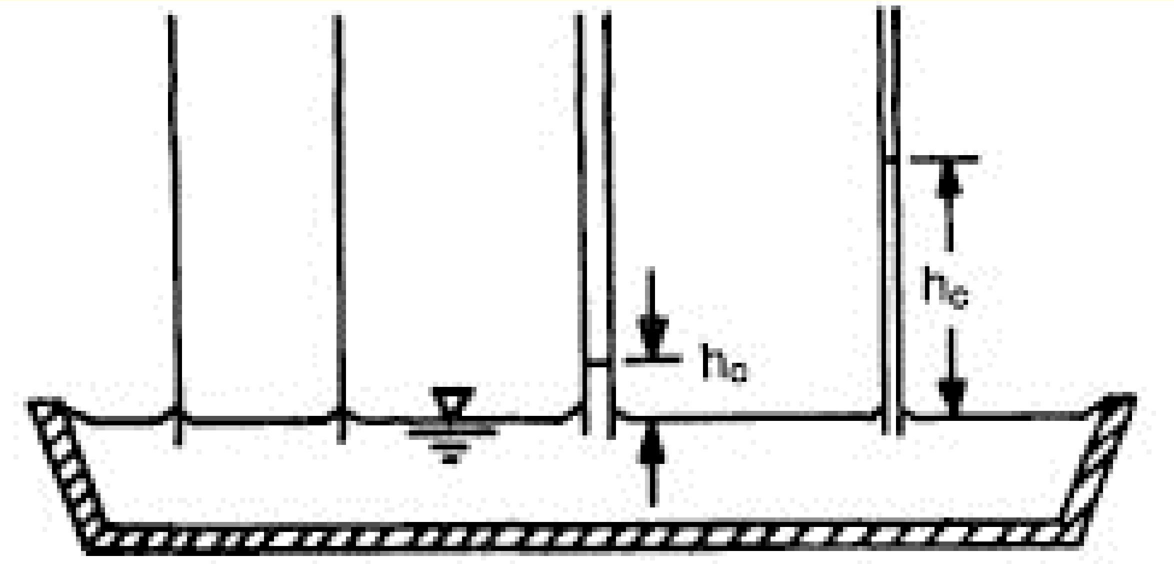
Capillarity in Soils

- A more complex phenomenon in soils due to complex geometry of particles
- More pronounced in fine grained soils
- Volume changes in fine grained soils can be in part attributed to capillarity
- Area of soil above water table where capillary rise is taking place is called the vadose zone
- Compressive stresses take place in the soil in an area with capillary rise takes place

Capillarity and Capillary Rise in Soils

- Equilibrium of capillary rise force and water head in a tube (z_c = height of water column)

$$\pi d T_s \cos\alpha = \frac{\pi z_c d^2 \gamma_w}{4}$$



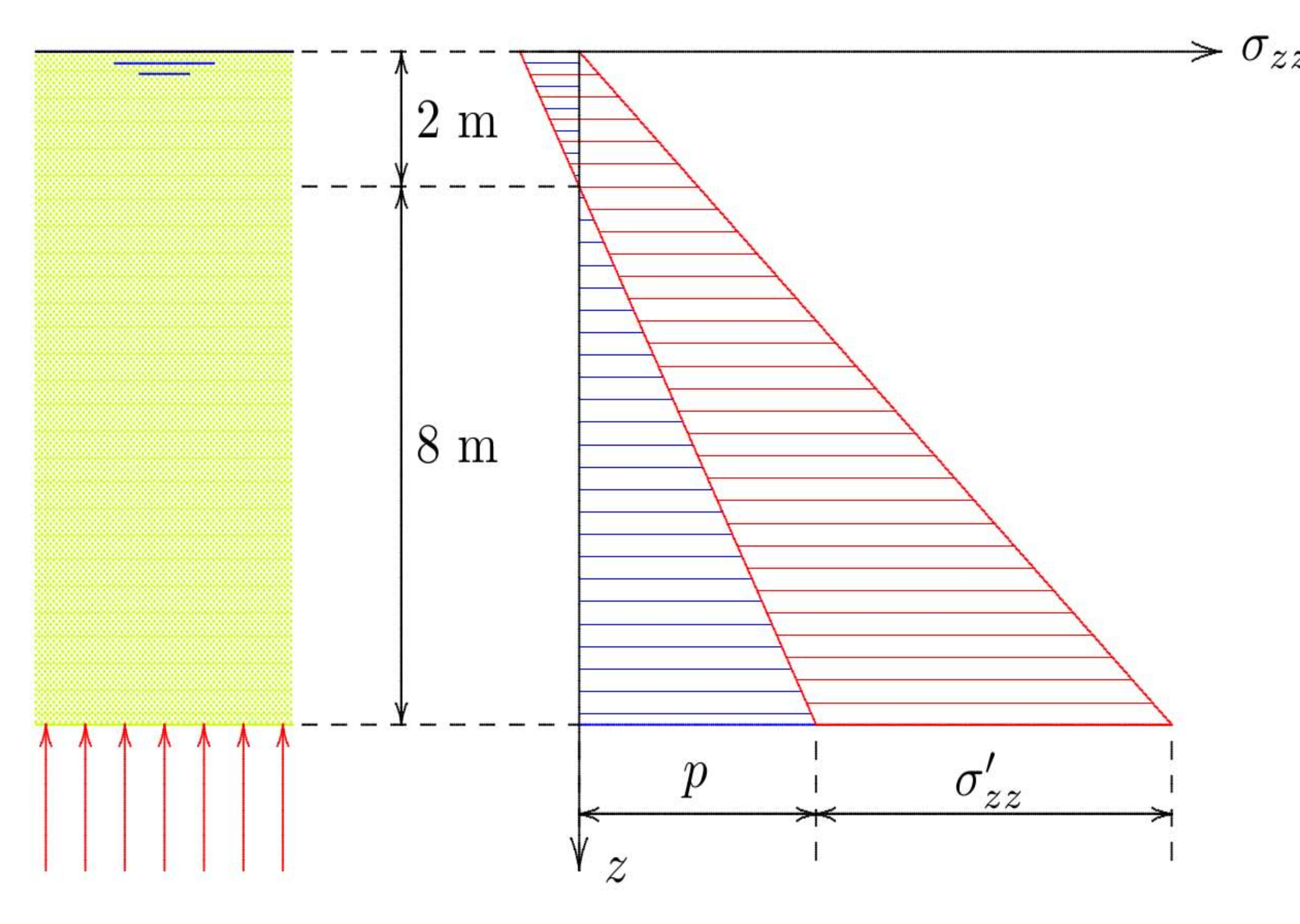
Capillarity and Capillary Rise

- Assumptions
  - $T_s = 0.0075$ g/mm
  - $\alpha = 0$
  - $d = D_{10}/5$ (D_{10} usually expressed in millimetres)
  - $\gamma_w = .001$ g/mm^3
- Substituting and Solving for z_c (millimetres)

$$z_c = \frac{150}{D_{10}}$$

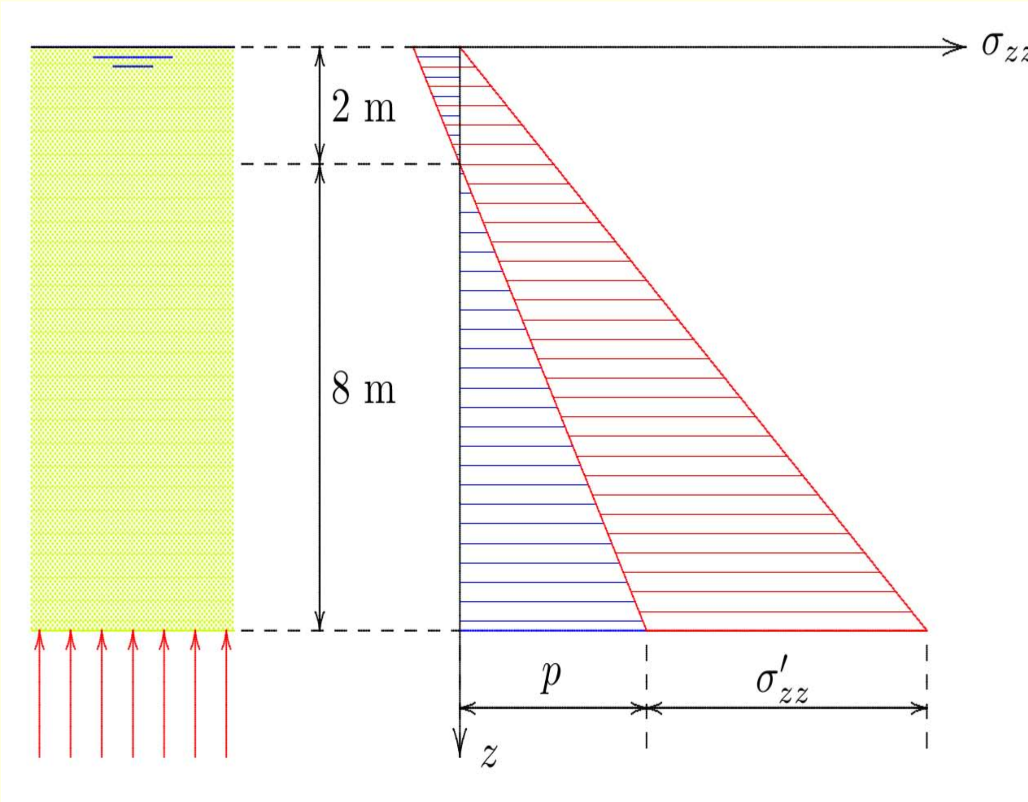
Capillarity Example

- Given
  - Soil, D_{10} = 0.074 mm (#200 sieve opening)
- Find
  - Capillary Rise in Soil
- Solution
  - z_c = 150/0.074 ~ 2027 mm ~ 2 m



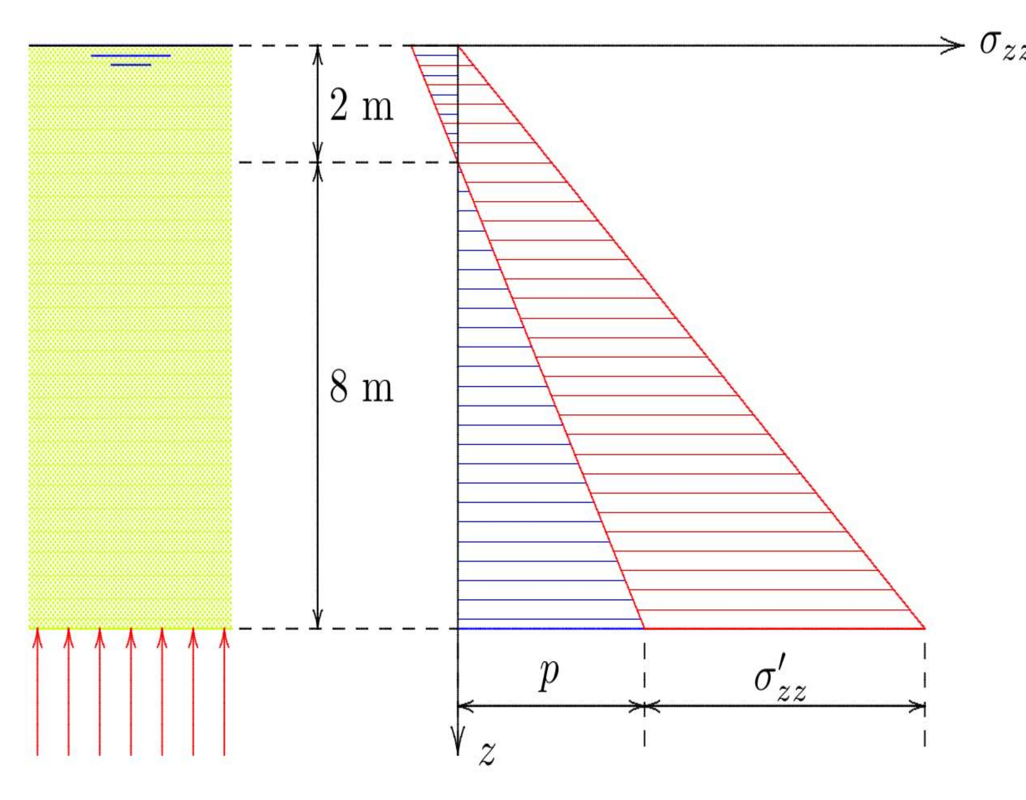
Capillary Rise and Effective Stress

- Note the following:
  - Phreatic surface is where the pore water pressure is zero
  - At the surface, since there is no surcharge, total stress is zero
  - Thus, the pore water pressure above the phreatic surface is negative with capillary rise, with the same slope as below the phreatic surface
  - There is thus a positive effective stress at the surface to balance the negative pore water pressure



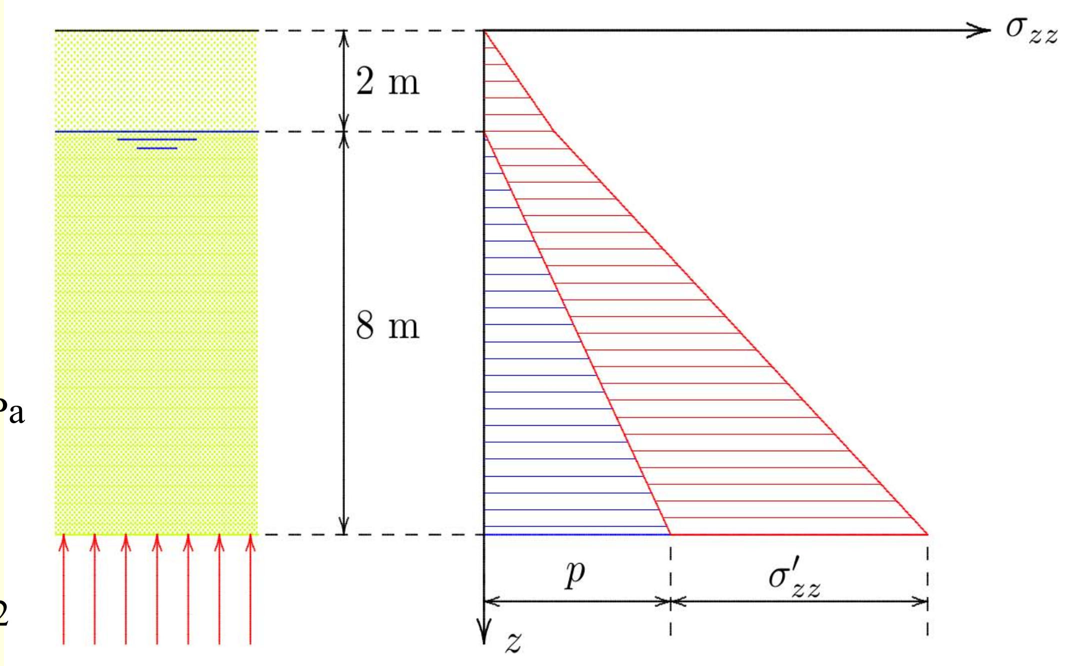
Capillary Rise and Effective Stress

- Assume that saturated unit weight = 20 kN/m^3
- At surface
  - Pore water pressure = -(2)(9.8) = -19.6 kPa
  - Total Stress = 0 kPa
  - Effective stress = 0 + 19.6 = 19.6 kPa
- At 2 m below surface
  - Pore water pressure = 0
  - Total Stress = (20)(2) = 40 kPa
  - Effective Stress = 40 – 0 = 40 kPa
- At 10 m below surface
  - Pore water pressure = (8)(9.8) = 78.4 kPa
  - Total Stress = (20)(10) = 200 kPa
  - Effective Stress = 200 – 78.4 = 121.6 kPa



Effective Stress without Capillary Rise

- Assume that saturated unit weight = 20 kN/m^3 and dry unit weight = 16 kN/m^3
- At surface
  - Pore water pressure = 0 kPa
  - Effective stress = 0 kPa
  - Total Stress = 0 kPa
- At 2 m below surface
  - Pore water pressure = 0
  - Total Stress = (16)(2) = 32 kPa
  - Effective Stress = 32 – 0 = 32 kPa
- At 10 m below surface
  - Pore water pressure = (8)(9.8) = 78.4 kPa
  - Total Stress = 32 + (20)(8) = 192 kPa
  - Effective Stress = 192 – 78.4 = 113.6 kPa



Questions?